

# STUDY OF LASER PLASMA INTERACTIONS USING AN EULERIAN VLASOV CODE

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D. J. Strozzi<sup>\*1</sup>, M. M. Shoucri<sup>†</sup>, A. Bers\*

*\*Plasma Science and Fusion Center  
Massachusetts Institute of Technology  
Cambridge, MA 02139*

*†Institut de Recherche de l'Hydro Québec  
Varennes, Canada*

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<sup>1</sup>email: dstrozzi@mit.edu. Poster available at [www.mit.edu/~dstrozzi](http://www.mit.edu/~dstrozzi)

# Abstract

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We use a one-dimensional Eulerian Vlasov Code to study laser-plasma interactions. The excitation of parametric instabilities, such as stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS) are an important issue for the efficiency of inertial confinement fusion. Our aim is to understand the growth, saturation, and coupling of these instabilities, in particular the kinetic aspects of these phenomena.

The code evolves both species with the fully relativistic Vlasov equation in the direction of laser propagation. It also incorporates a cold fluid transverse velocity for each species, which is needed to couple the transverse and longitudinal dynamics. The code uses the method of fractional steps and cubic spline interpolation to advance the distribution function in time.

The plasma is finite and not periodic. On the boundaries we place “absorbing plates” that collect the particles flowing onto them. This charge enters Poisson’s equation as a boundary condition on the electric field.

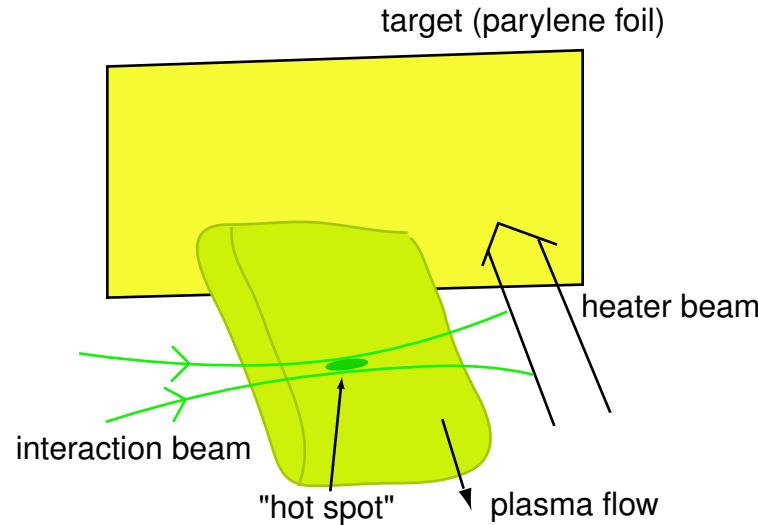
We are able to run the code for many times longer than it takes the laser to cross the computational domain. Once the laser passes a region of plasma, it modifies the equilibrium. This modification is sufficient for SRS to grow from; we do not “seed” SRS by imposing any perturbations or a secondary laser.

For parameters similar to the recent Trident single hot spot experiments, SRS is not limited by the Langmuir Decay Instability (LDI) or other ion processes, but by kinetic effects. The code clearly resolves the formation of large vortices in phase-space due to electron trapping in the field of the SRS electron plasma wave. We also see SBS starting to develop, but we have not run for long enough times for the ions to move through several ion-acoustic wave periods.

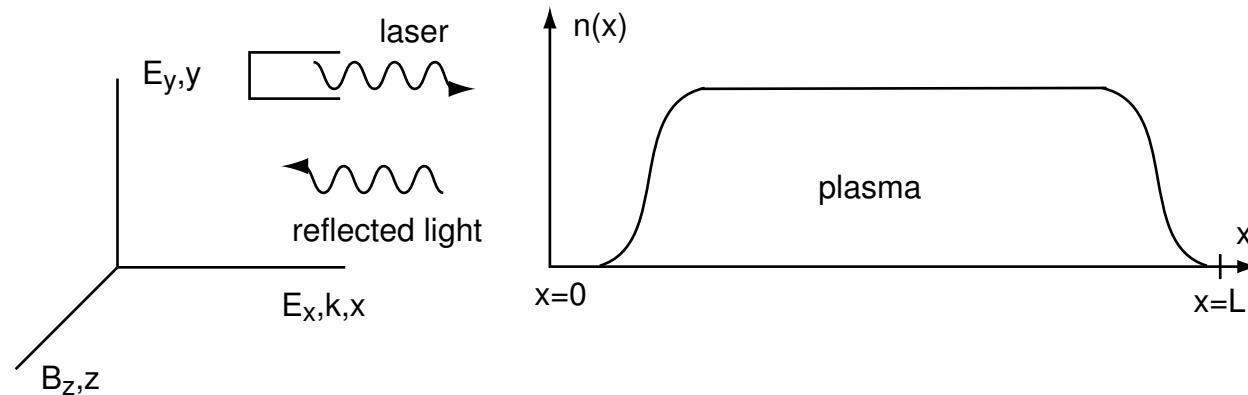
Work is underway to extend the code to  $1\frac{1}{2}$  dimensions and to parallelize it.

# Single Hot Spot Experiments

[D. S. Montgomery, J. A. Cobble, et al. *Phys. Plasmas*, 9(5):2311–2320, 2002.]



## One-Dimensional Model Geometry



# 1-Dimensional Kinetic Model

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- Spatial variation only in  $x$ :  $\nabla \rightarrow \partial/\partial x$
- Fields:  $\vec{E} = (E_x, E_y, 0)$   $\vec{B} = (0, 0, B_z)$
- Transverse cold beam:  $F_s(x, p_x, P_y, t) = \delta(P_y)f_s(x, p_x, t)$ ,  $P_y = m_s v_{ys} + q_s A_y = y$  canonical mom.  $y$  momentum equation becomes [Gabor, Proc. IRE 33:792, 1945]

$$\boxed{\frac{\partial v_{ys}}{\partial t} = \frac{q_s}{m_s} E_y}$$

- Longitudinal dynamics: Relativistic 1-D Vlasov Eqn. for  $f_s(x, p_x, t)$

$$\boxed{\frac{\partial f_s}{\partial t} + v_x \frac{\partial f_s}{\partial x} + \frac{q_s}{m_s} (E_x + v_{ys} B_z) \frac{\partial f_s}{\partial p_x} = 0}$$

- Longitudinal field:  $E_x$  (electrostatic perturbations)

Poisson  $\rightarrow$  
$$\boxed{\frac{\partial E_x}{\partial x} = \frac{e}{\epsilon_0} (n_i - n_e)}$$

- Transverse fields: laser, electromagnetic perturbations

Ampère  $\rightarrow$  
$$\frac{\partial E_y}{\partial t} + c^2 \frac{\partial B_z}{\partial x} = -\frac{1}{\epsilon_0} J_y$$

$$\boxed{E^\pm \equiv E_y \pm c B_z \quad \rightarrow \quad \left( \frac{\partial}{\partial t} \pm c \frac{\partial}{\partial x} \right) E^\pm = -\frac{1}{\epsilon_0} J_y}$$

# The Time-Stepping Algorithm

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Operator splitting with cubic spline for Vlasov; leapfrog for electromagnetic fields [Ghizzo, Bertrand, Shoucri *et al.*, *Journ. Comp. Phys.* 90(431), 1990]

$t/dt$

0

$0^+$

$\frac{1}{2}^-$

$\frac{1}{2}$

$\frac{1}{2}^+$

$1^-$

1

$1^+$

$$(\text{leapfrog}) \quad f \xrightarrow{\substack{(\partial_t + v_x \partial_x) f = 0 \\ f_0}} f_{\frac{1}{2}^-} \xrightarrow{\substack{(\partial_t + F_{1/2} \partial_{p_x}) f = 0, \\ F = q(E_x + v_y B_z)}} f_{\frac{1}{2}^+} \xrightarrow{(\partial_t + v_x \partial_x) f = 0} f_1$$

$$(\text{what we do}) \quad f \xrightarrow{(\partial_t + v_x \partial_x) f = 0} f_{1^-} \xrightarrow{(\partial_t + F_{1/2} \partial_{p_x}) f = 0} f_1$$

$$n \quad n_0 \quad n_{\frac{1}{2}} = \frac{1}{2}(n_0 + n_1) \quad n_1[f_{1^-}]$$

$$N_p \quad N_{p0} \quad N_{p\frac{1}{2}} = \frac{1}{2}(N_{p0} + N_{p1}) \quad N_{p1}[n_1]$$

$$dN_w \quad dN_{w0} \quad dN_{w\frac{1}{2}} = \frac{1}{2}(dN_{w0} + dN_{w1}) \quad dN_{w1}[f_{1^-}]$$

$$N_w \quad N_{w0} \quad N_{w\frac{1}{2}} = N_{w0} + \frac{1}{2}dN_{w\frac{1}{2}} \quad N_{w1} = N_{w0} + dN_{w1}$$

$$E_x \quad E_{x\frac{1}{2}}[n_{\frac{1}{2}}, N_{p\frac{1}{2}}, N_{w\frac{1}{2}}]$$

$$E^\pm \xrightarrow{(\partial_t \pm \partial_x) E^\pm = 0} E_{\frac{1}{2}}^\pm = \frac{1}{2}(E_0^\pm + E_{1^-}^\pm) \xrightarrow{(\partial_t \pm \partial_x) E^\pm = -\epsilon_0^{-1} J_{y1}} E_{1^+}^\pm$$

$$v_y \quad v_{y0} \xrightarrow{m \partial_t v_y = q E_{y\frac{1}{2}}} v_{y\frac{1}{2}} = \frac{1}{2}(v_{y0} + v_{y1}) \xrightarrow{} v_{y1}$$

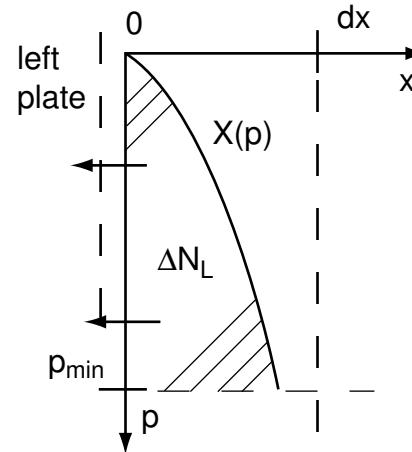
# Boundary Conditions

- Nonperiodic grids in  $x$  and  $p_x$ ; plasma is finite.
- “Absorbing plates” (transparent to electromagnetic fields) at  $x = 0$  and  $x = L$ . Particles that flow out to the plates stay there and no longer evolve.

$N_{sL}$ ,  $N_{sP}$ ,  $N_{sR} = \#$  of particles of species  $s$  on left plate, in plasma, on right plate

$$X(p) = -dt v(p) \quad v(p) = \frac{p}{\gamma m}$$

$$\begin{aligned} \Delta N_L &= \int_{p_{min}}^0 dp \int_0^{X(p)} dx f(x, p) \\ &\approx -dt \int_{p_{min}}^0 dp v f(0, p) \end{aligned}$$



- Charges on plates enter  $E_x$  as a boundary condition.

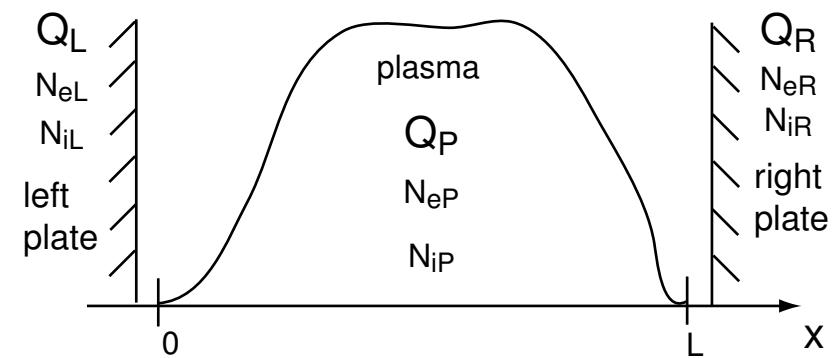
$$\frac{\partial E_x}{\partial x} = \epsilon_0^{-1} \rho$$

$$Q_L + Q_P + Q_R = 0$$

$$\therefore E_x(x < \text{left plate}) = 0$$

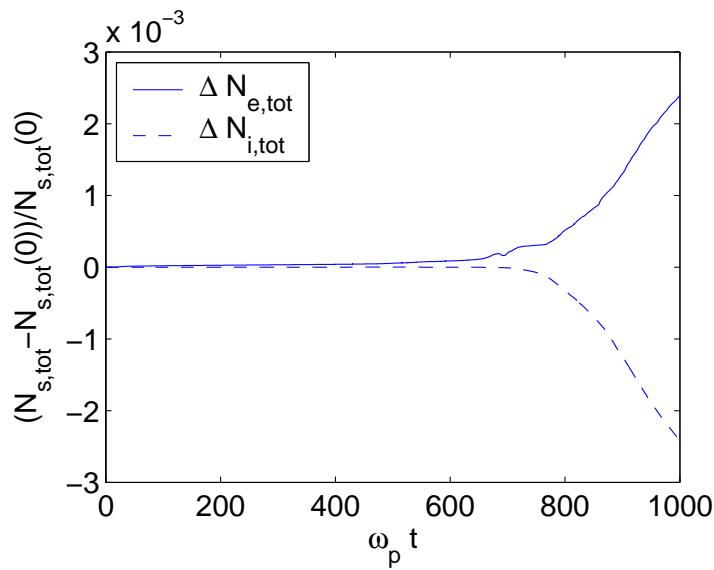
$$E(x = 0) = \frac{1}{2\epsilon_0} (Q_L - Q_P - Q_R)$$

$$E(x) = E(x = 0) + \epsilon_0^{-1} \int_0^x \rho(x') dx'$$

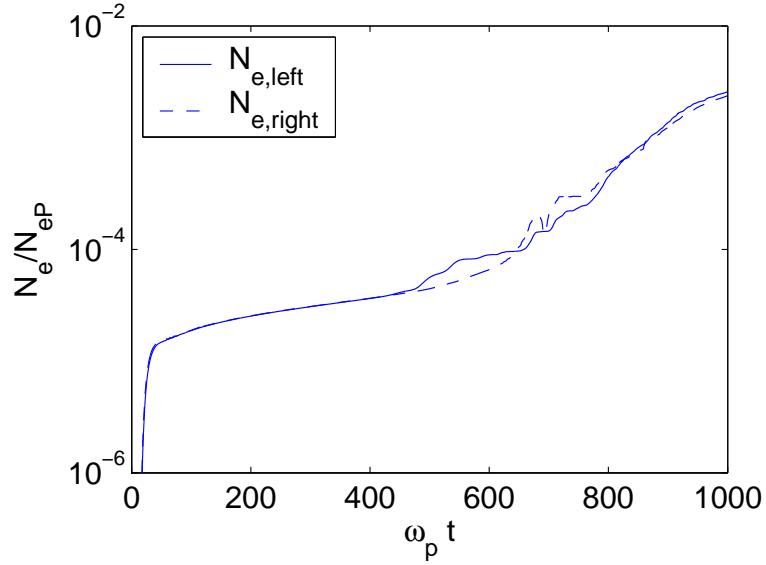


# Outflow to plates

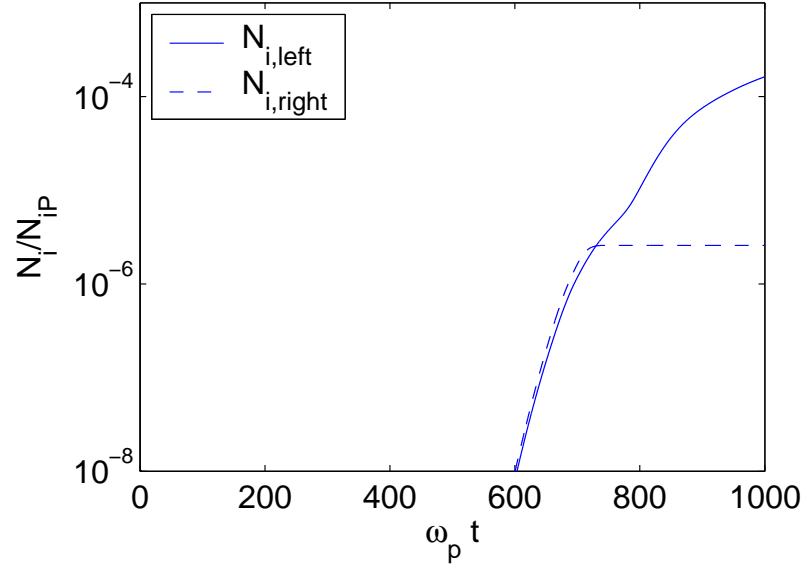
## Change in total number of particles



## Electrons on left, right walls



## Ions on left, right walls



# Run Parameters

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- Scales used in code:

$$\text{time: } t_0 = 1/\omega_p \quad \text{length: } d_e = c/\omega_p \quad \text{speed: } c \quad \text{momentum (all species): } p_0 = m_e c$$

- Plasma Parameters:

Plasma length:  $L = 75 \mu\text{m} = 160 d_e$

Plasma density:  $n_e = 1.28 \cdot 10^{20} \text{ cm}^{-3} = 0.032 n_{crit}$

initial electron Temperature:  $T_e = 350 \text{ eV}$       initial ion Temperature:  $T_i = 100 \text{ eV}$

electron thermal speed:  $v_{Te} = 0.0262 c$       ion thermal speed:  $v_{Ti} = 3.27 \cdot 10^{-4} c$

Plasma frequency:  $\omega_p = 6.4 \cdot 10^{14} \text{ rad/s} \rightarrow t_0 = 1.56 \cdot 10^{-15} \text{ s}, d_e = 470 \text{ nm}$

Debye length:  $\lambda_{De} = 12.3 \text{ nm} = 0.0262 d_e$

- Laser Parameters:

Laser frequency:  $\omega_0 = 3.57 \cdot 10^{15} \text{ rad/s} = 5.59 \omega_p$       free-space wavelength:  $\lambda_0 = 527 \text{ nm} = 1.12 d_e$

Laser intensity:  $I_0 = 10^{16} \text{ W/cm}^2$

electron quiver velocity:  $v_{quiv} = .045 c$

- Numerical parameters:

Timestep:  $dt = c dx = 0.01/\omega_p$

Total runtime:  $T = 1.56 \text{ ps} = 1000 t_0$

Spatial gridpoints:  $16001 \rightarrow dx = 0.01 d_e$

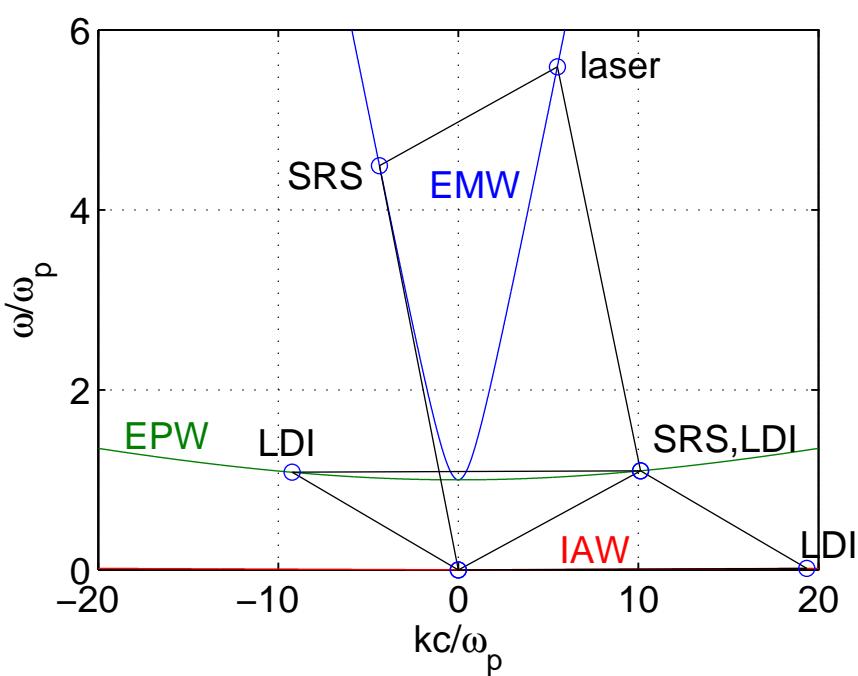
electron momentum gridpoints:  $301 \rightarrow dp_e = 0.0027 m_e c = 0.102 p_{Te}$

ion momentum gridpoints:  $151 \rightarrow dp_i = 0.069 m_e c = 0.187 p_{Ti}$

# Parametric Instabilities: Coupling of Modes

EPW (e <sup>-</sup> plasma)	$\omega^2 = \omega_{pe}^2 + 3v_{Te}^2 k^2$	e/s $\vec{E} \parallel \vec{k}$
IAW (ion acoustic)	$\omega^2 = c_a^2 k^2$	e/s $\vec{E} \parallel \vec{k}$
EMW (electromagnetic)	$\omega^2 = \omega_{pe}^2 + c^2 k^2$	e/m $\vec{E} \perp \vec{k}$

Matching:  $\omega_1 = \omega_2 + \omega_3$      $k_1 = k_2 + k_3$



$$\vec{E}_j = a_j(x, t) \sqrt{\frac{2\omega_j}{\epsilon_0}} \hat{\epsilon}_j e^{(k_j x - \omega_j t)}$$

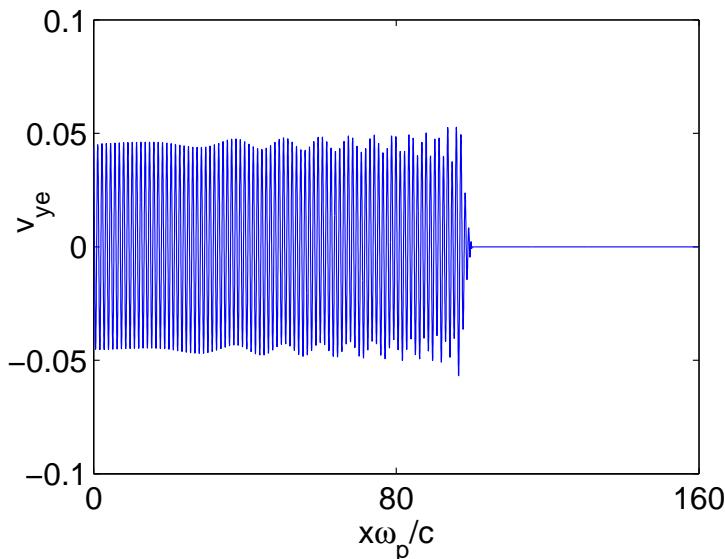
$$|a_j|^2 = \frac{\epsilon_0}{2\omega_j} |E_j|^2 = \text{action density}$$

$$\begin{aligned}\frac{D a_1}{D_1 t} &= -K a_2 a_3 \\ \frac{D a_2}{D_2 t} &= K^* a_1 a_3^* \\ \frac{D a_3}{D_3 t} &= K^* a_1 a_2^*\end{aligned}$$

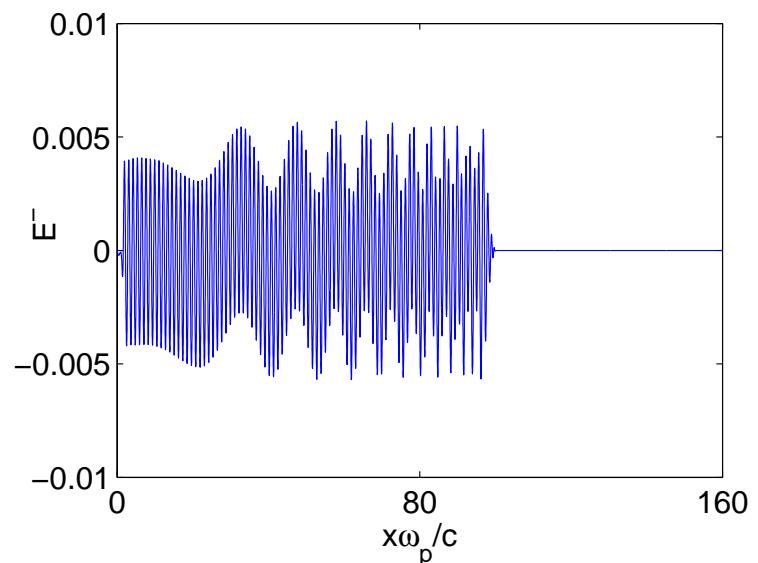
$$\frac{D}{D_j t} = \frac{\partial}{\partial t} + v_{gj} \frac{\partial}{\partial x} + \nu_j$$

# Laser-Modified Equilibrium: Electromagnetic Fields

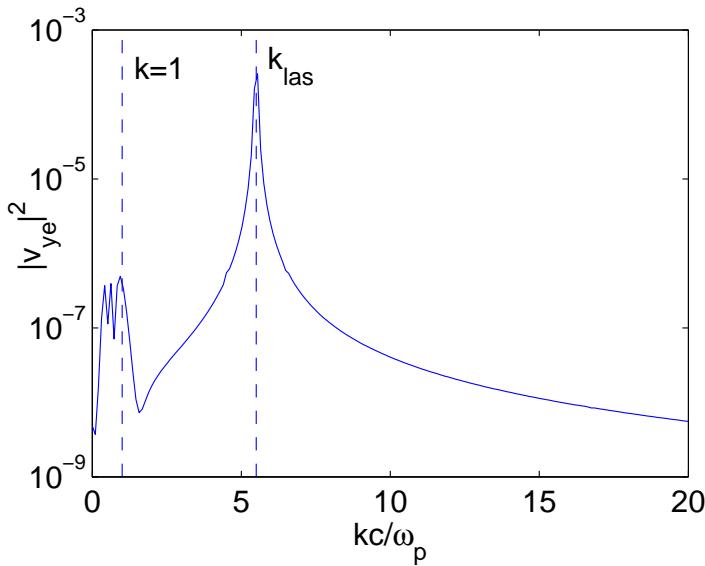
$v_{ye}$  at  $\omega_p t = 100$



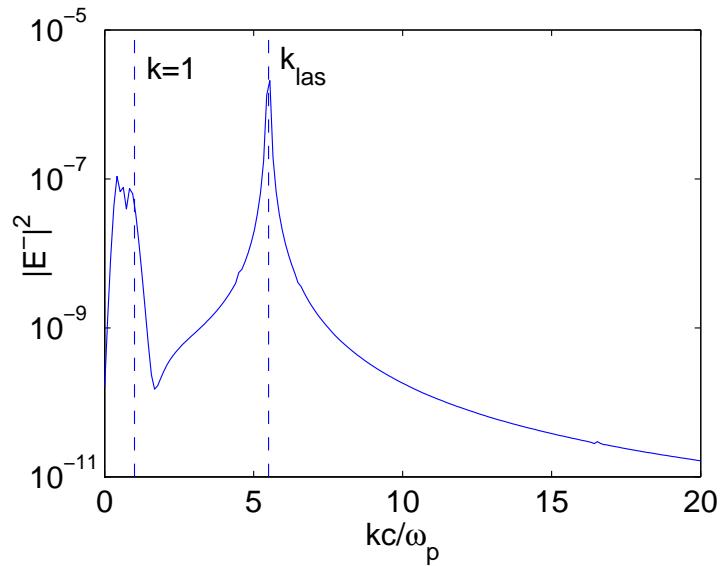
$E^-$  at  $\omega_p t = 100$



$|v_{ye}(\omega)|^2$  at  $\omega_p t = 100, x\omega_p/c = 20 - 80$

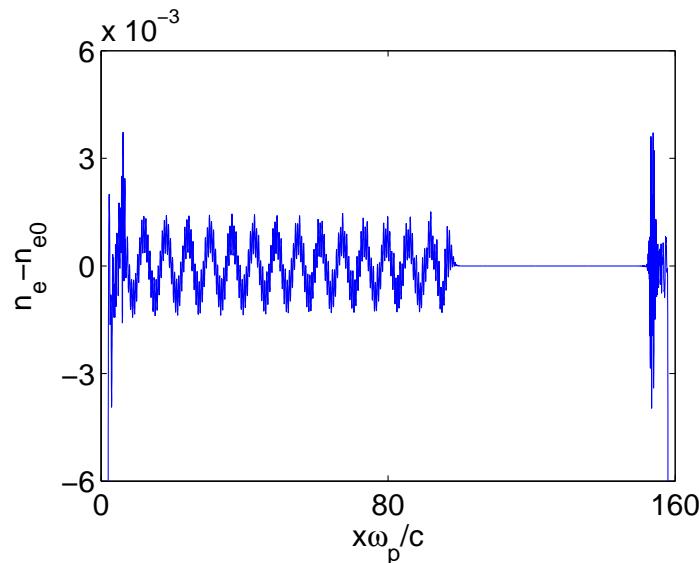


$|E^-(\omega)|^2$  at  $\omega_p t = 100, x\omega_p/c = 20 - 80$

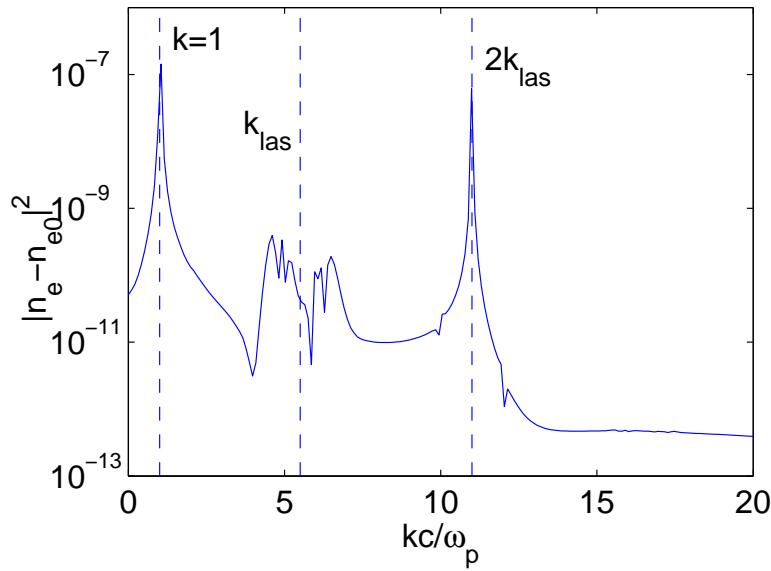


# Laser-Modified Equilibrium: Electrostatic Fields

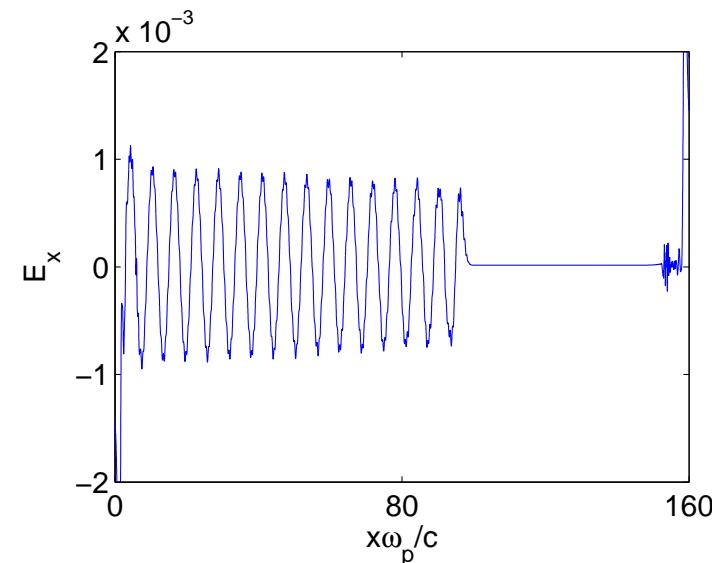
$n_e - n_{e0}$  at  $\omega_p t = 100$



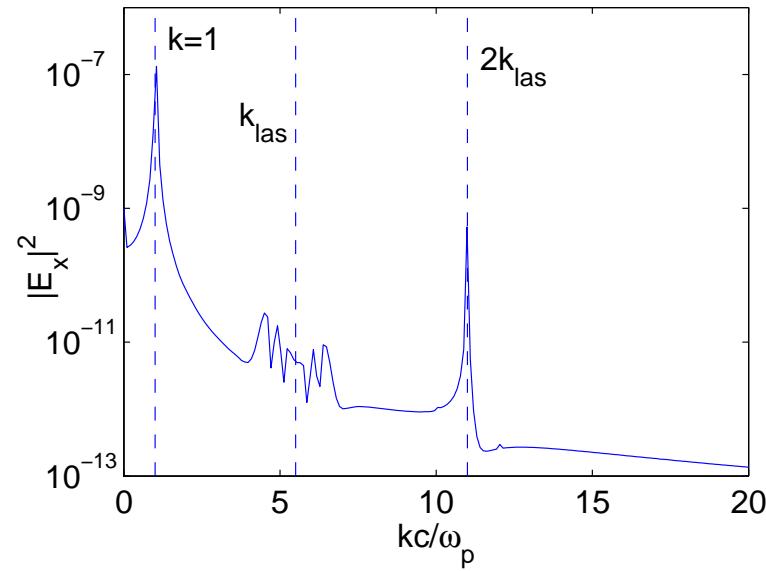
$|n_e(\omega)|^2$  at  $\omega_p t = 100$ ,  $x\omega_p/c = 20 - 80$



$E_x$  at  $\omega_p t = 100$

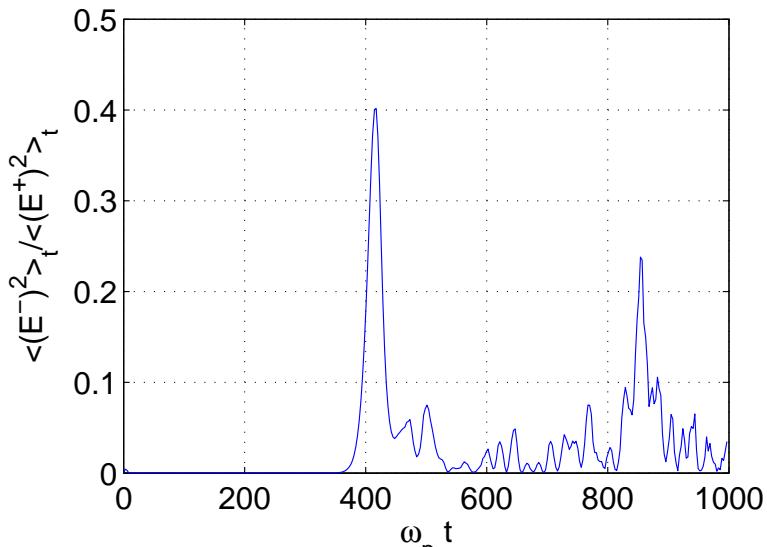


$|E_x(\omega)|^2$  at  $\omega_p t = 100$ ,  $x\omega_p/c = 20 - 80$

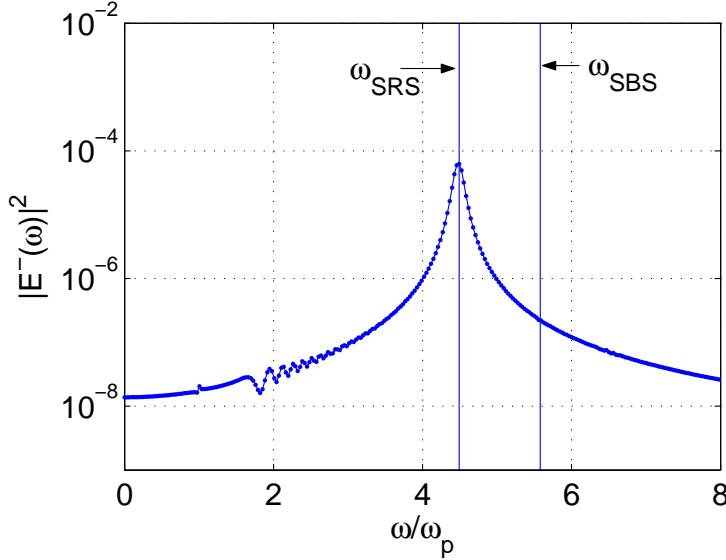


# Reflected Light at $x = 0$

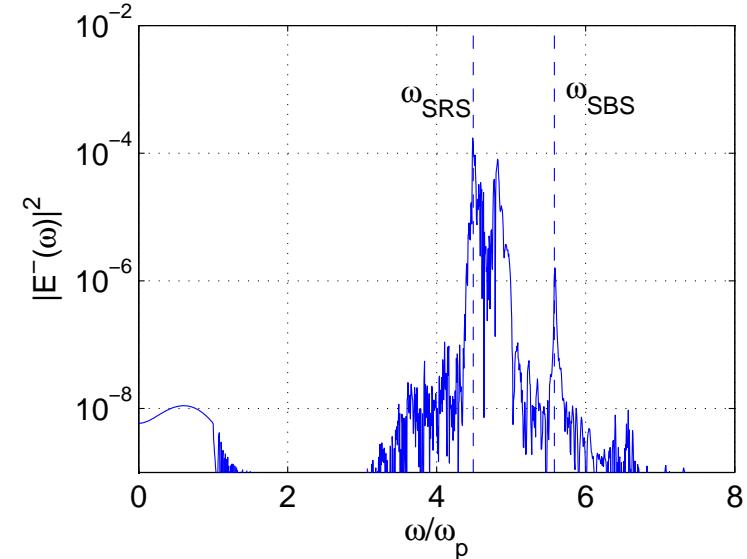
Instantaneous Reflectivity (Avg.=3.1%)



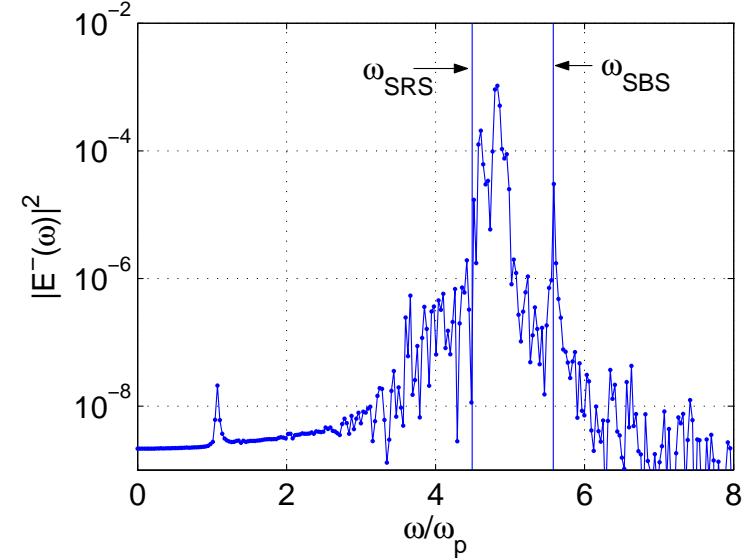
$|E^-(\omega)^2|$  at  $x = 0$ ,  $t\omega_p = 200 - 400$



$|E^-(\omega)^2|$  at  $x = 0$ ,  $t\omega_p = 0 - 1000$

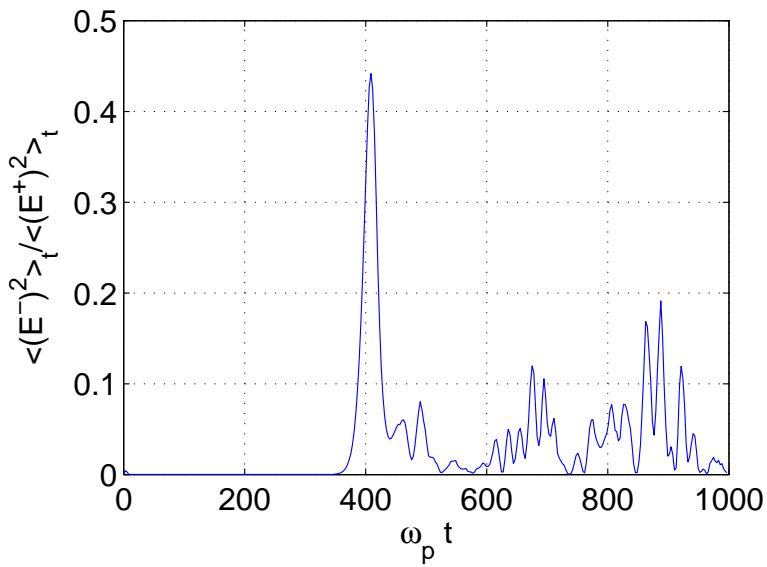


$|E^-(\omega)^2|$  at  $x = 0$ ,  $t\omega_p = 800 - 1000$

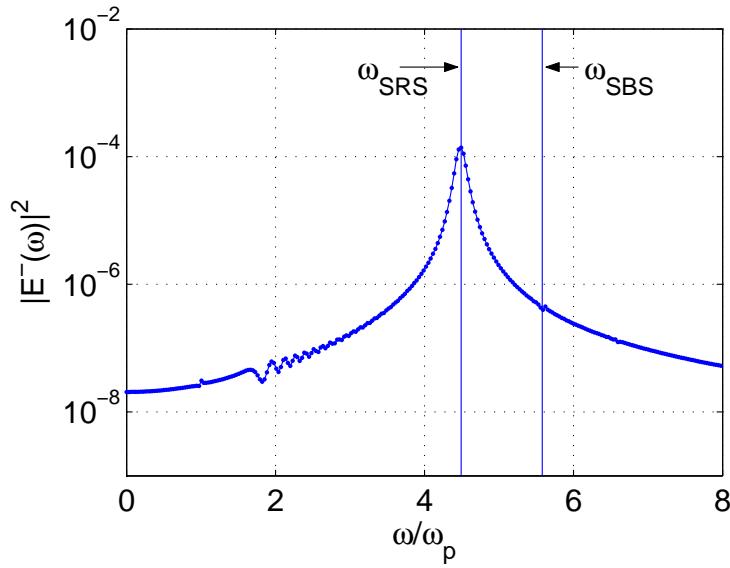


# Reflected Light at $x = 0$ , Stationary Ions

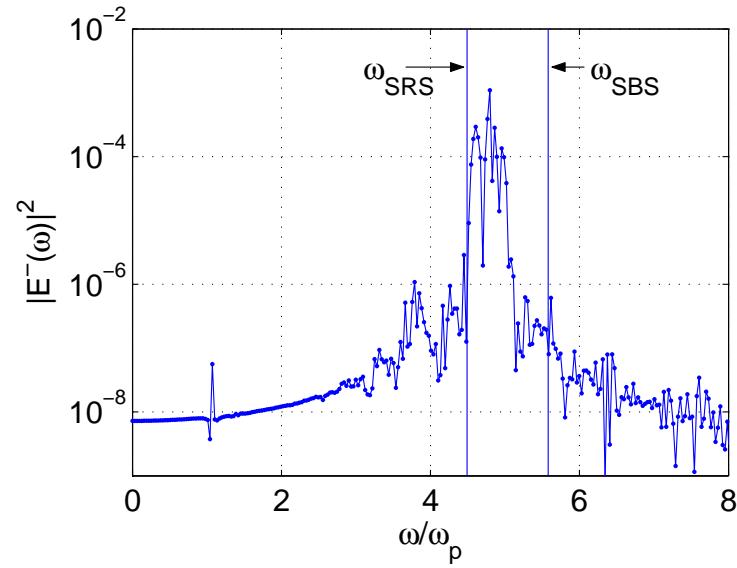
Instantaneous Reflectivity (Avg.: 3.4%)



$|E^-(\omega)^2|$  at  $x = 0$ ,  $t\omega_p = 200 - 400$

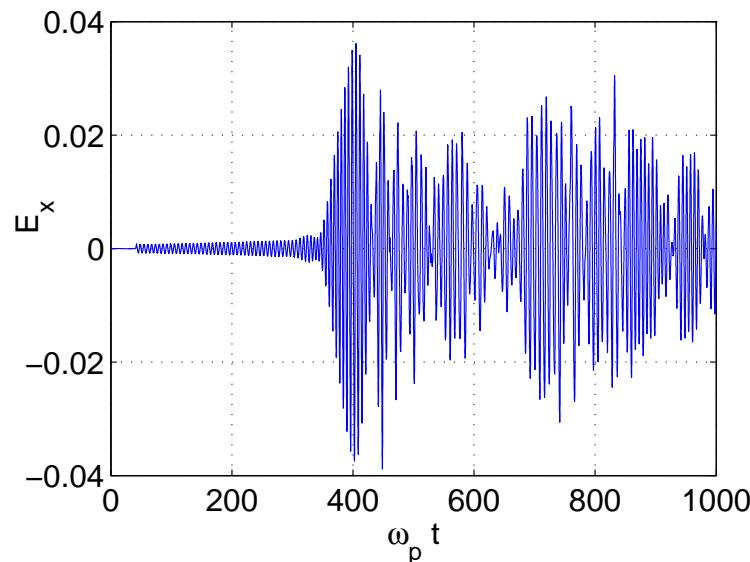


$|E^-(\omega)^2|$  at  $x = 0$ ,  $t\omega_p = 800 - 1000$

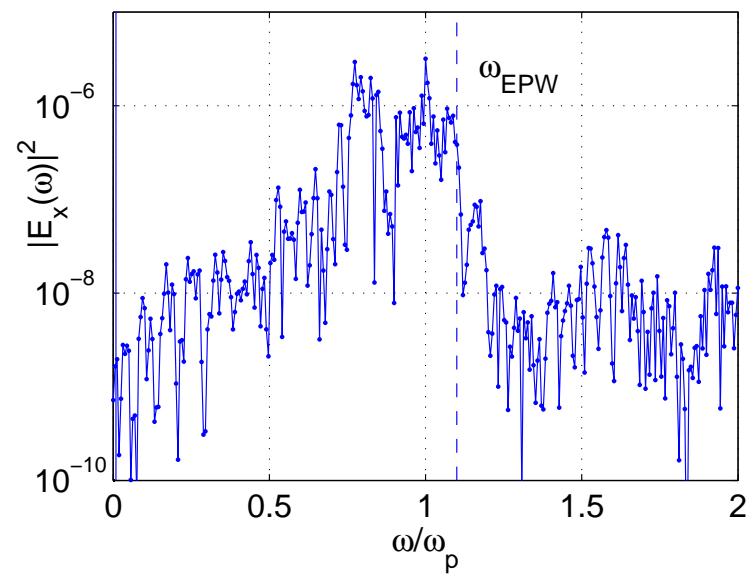


# Longitudinal Electric Field $E_x$ vs. $t$

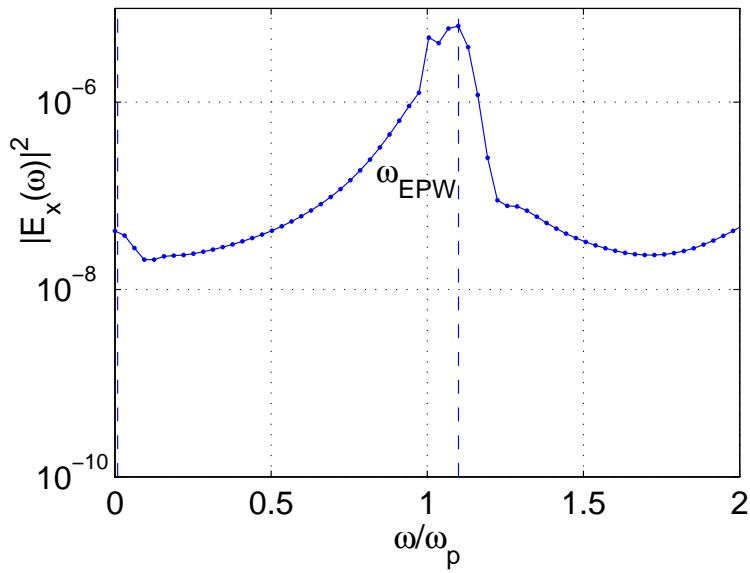
$E_x$  at  $x\omega_p/c = 40$



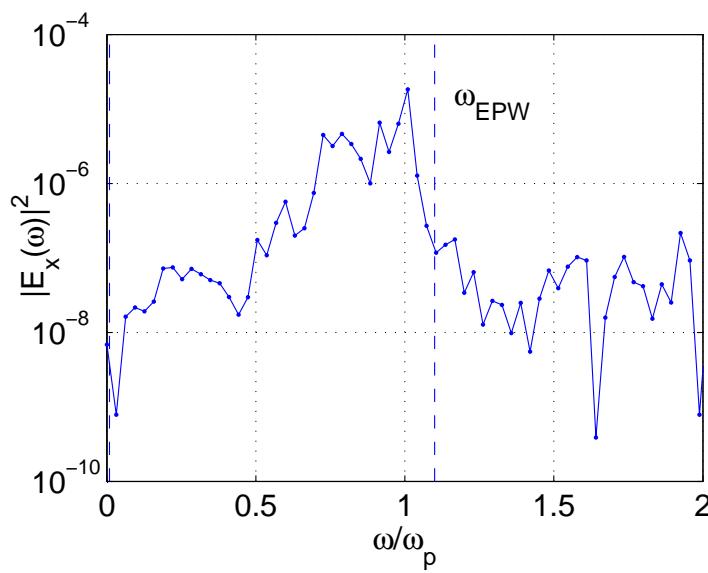
$|E_x(\omega)|^2$  at  $x\omega_p/c = 40, \omega_p t = 0 - 1000$



$|E_x(\omega)|^2$  at  $x\omega_p/c = 40, \omega_p t = 200 - 400$

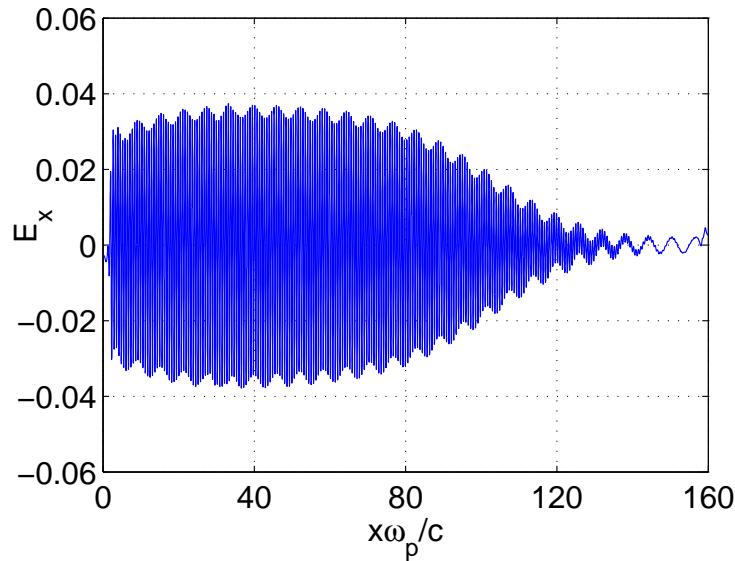


$|E_x(\omega)|^2$  at  $x\omega_p/c = 40, \omega_p t = 800 - 1000$

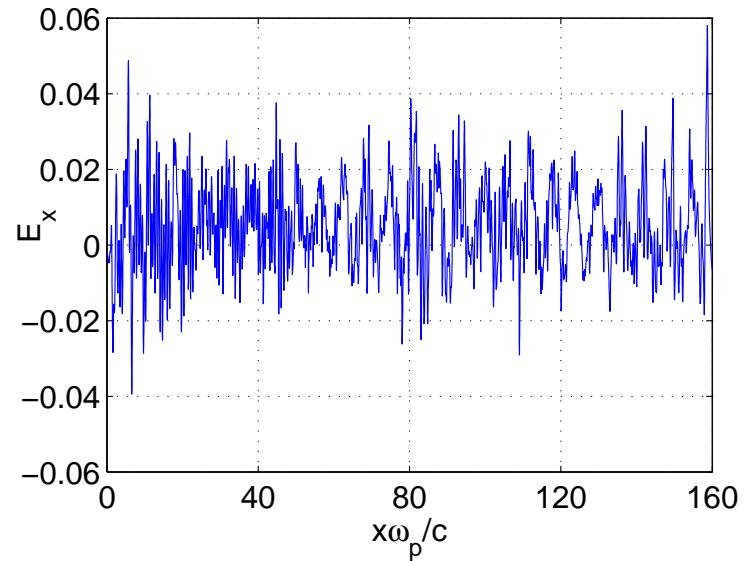


# Longitudinal Electric Field $E_x$ vs. $x$

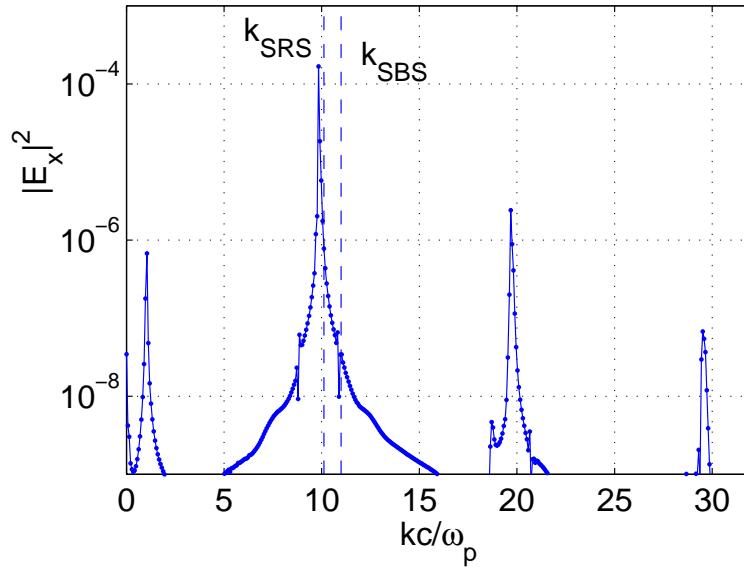
$E_x$  at  $\omega_p t = 400$



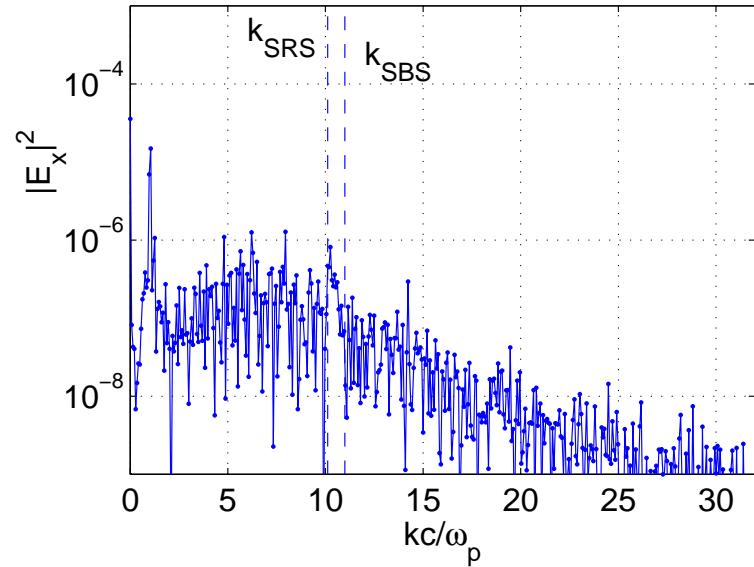
$E_x$  at  $\omega_p t = 900$



$|E_x(k)|^2$  at  $\omega_p t = 400, x\omega_p/c = 30 - 120$

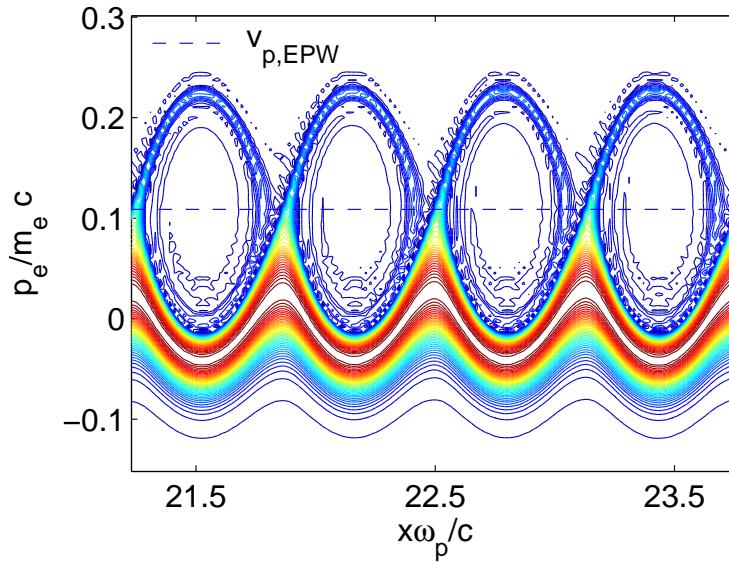


$|E_x(k)|^2$  at  $\omega_p t = 900, x\omega_p/c = 30 - 120$

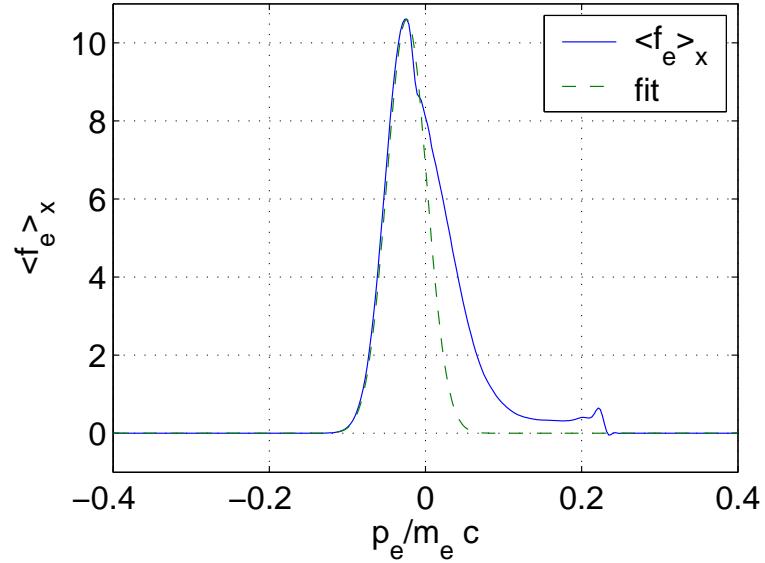


# Trapped $f_e$ during first SRS burst

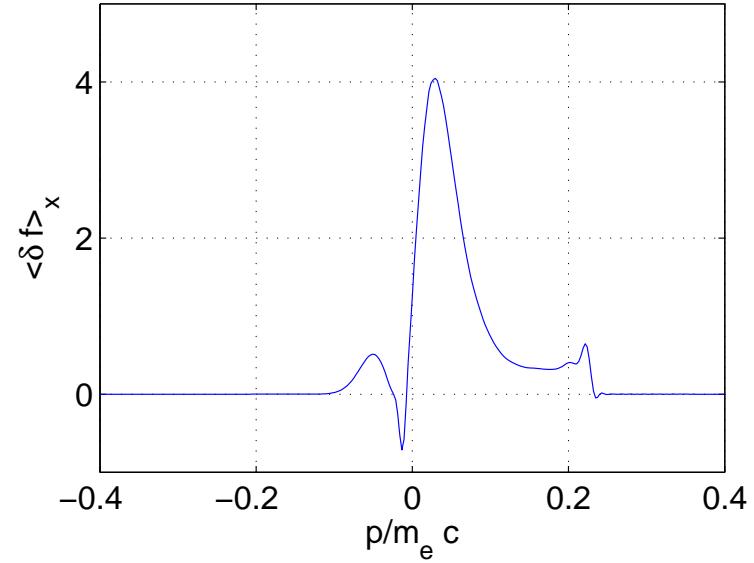
Contour Plot of  $f_e$  at  $\omega_p t = 400$



Spatially-averaged  $\langle f_e \rangle_x$

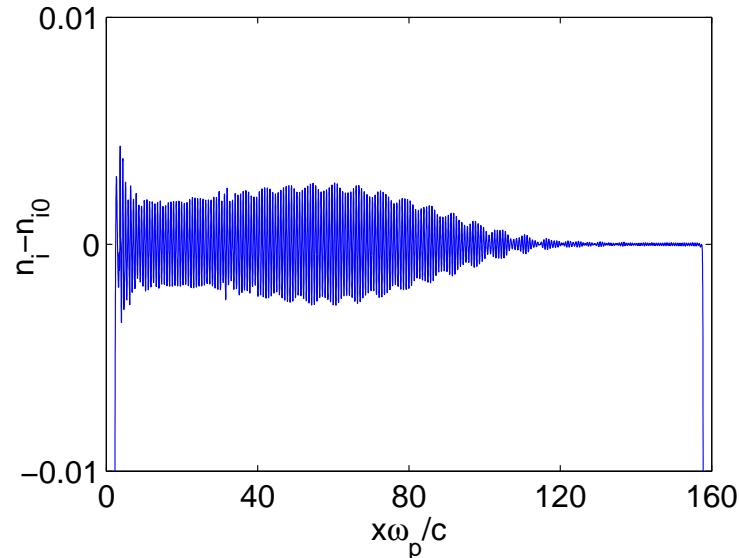


$\langle f_e \rangle_x - \text{Maxwellian fit}$

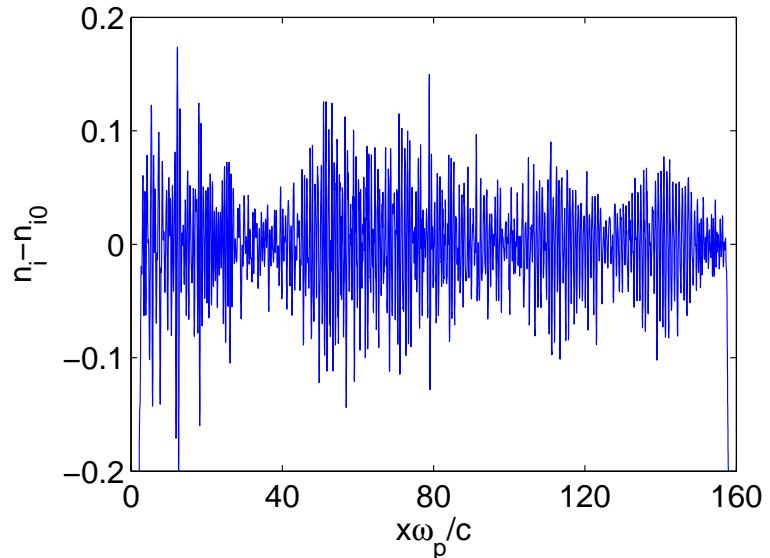


# Ion Density Fluctuations vs. $x$

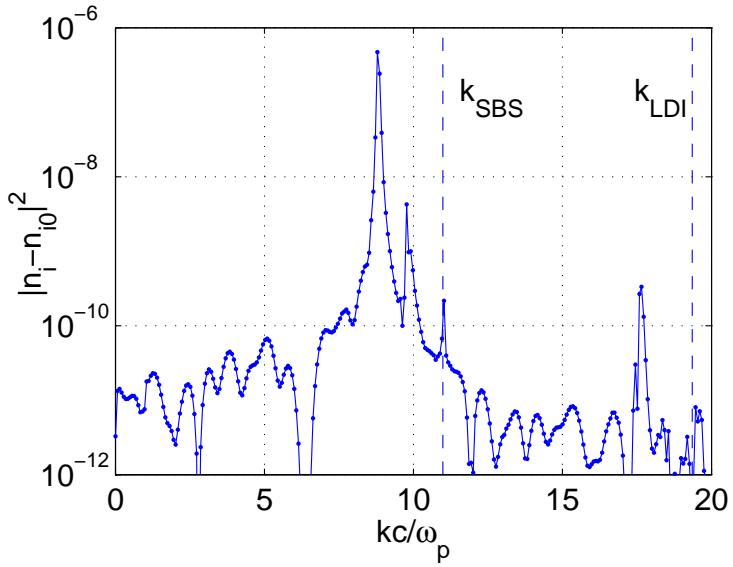
$n_i - n_{i0}$  at  $\omega_p t = 450$



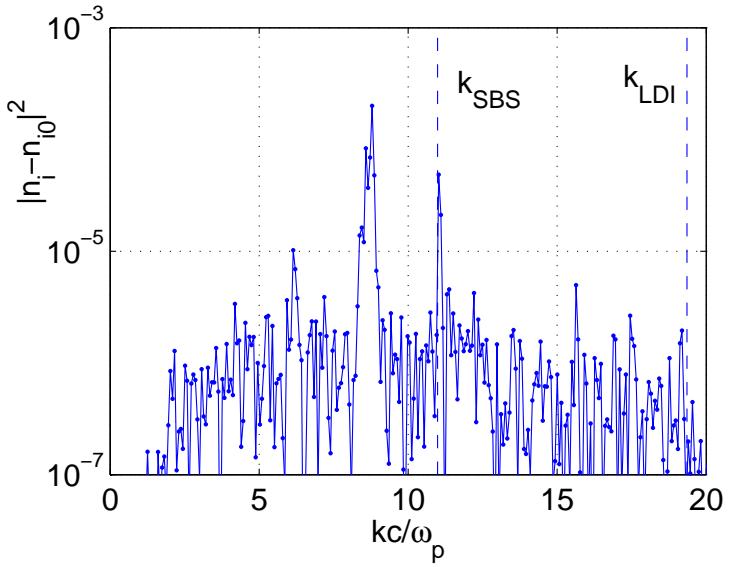
$n_i - n_{i0}$  at  $\omega_p t = 1000$



$|n_i(\omega)|^2$  at  $\omega_p t = 450, x\omega_p/c = 30 - 120$

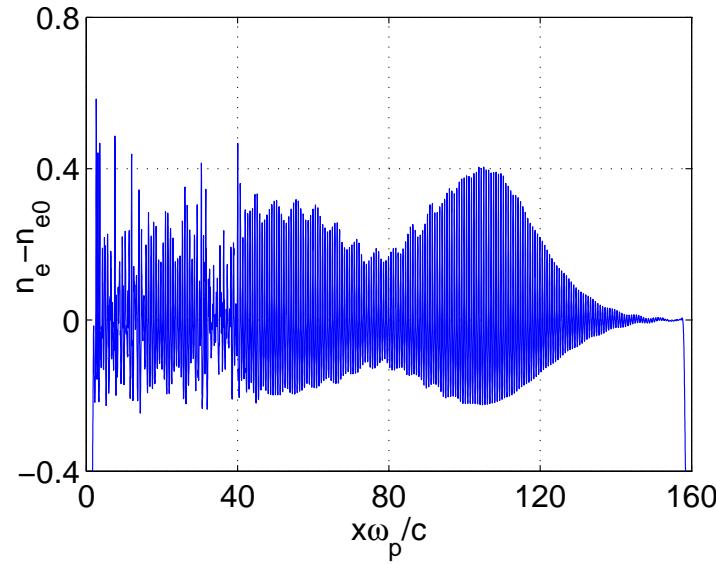


$|n_i(\omega)|^2$  at  $\omega_p t = 1000, x\omega_p/c = 30 - 120$

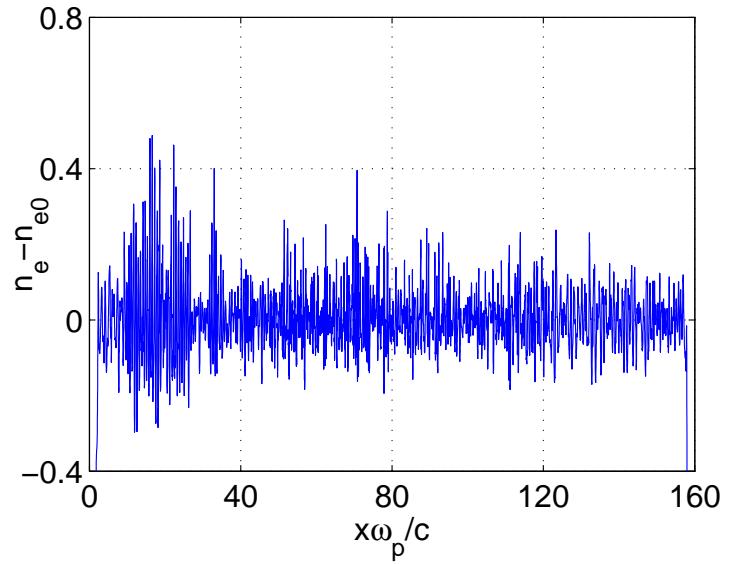


# Electron Density Fluctuations vs. $x$

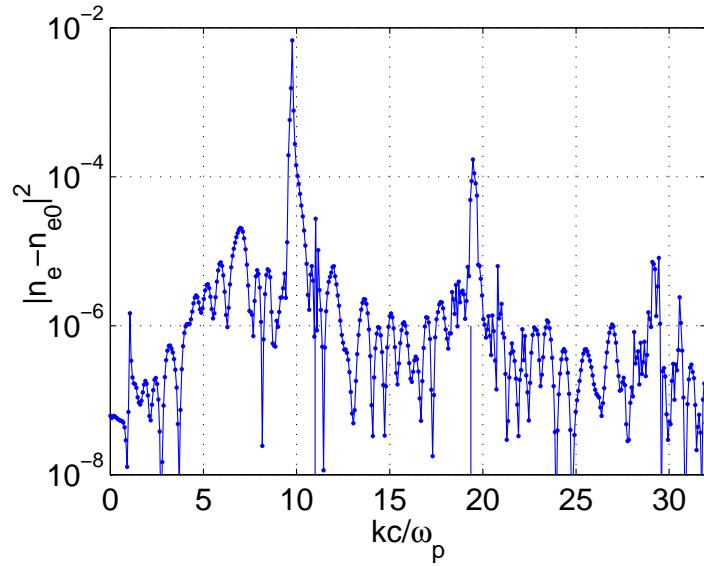
$n_e - n_{e0}$  at  $\omega_p t = 450$



$n_e - n_{e0}$  at  $\omega_p t = 1000$

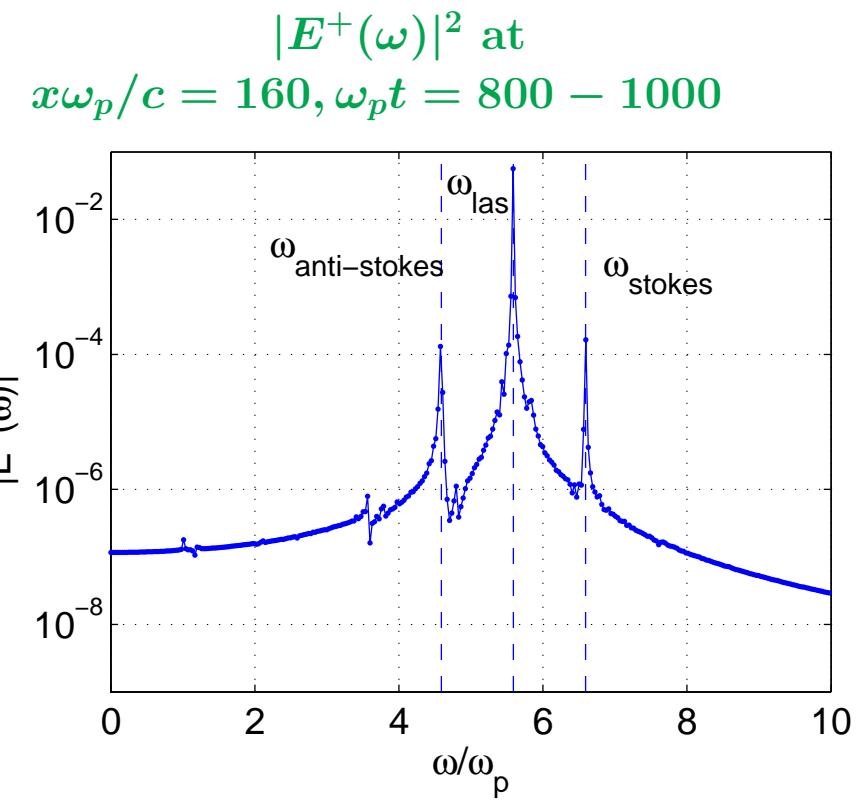
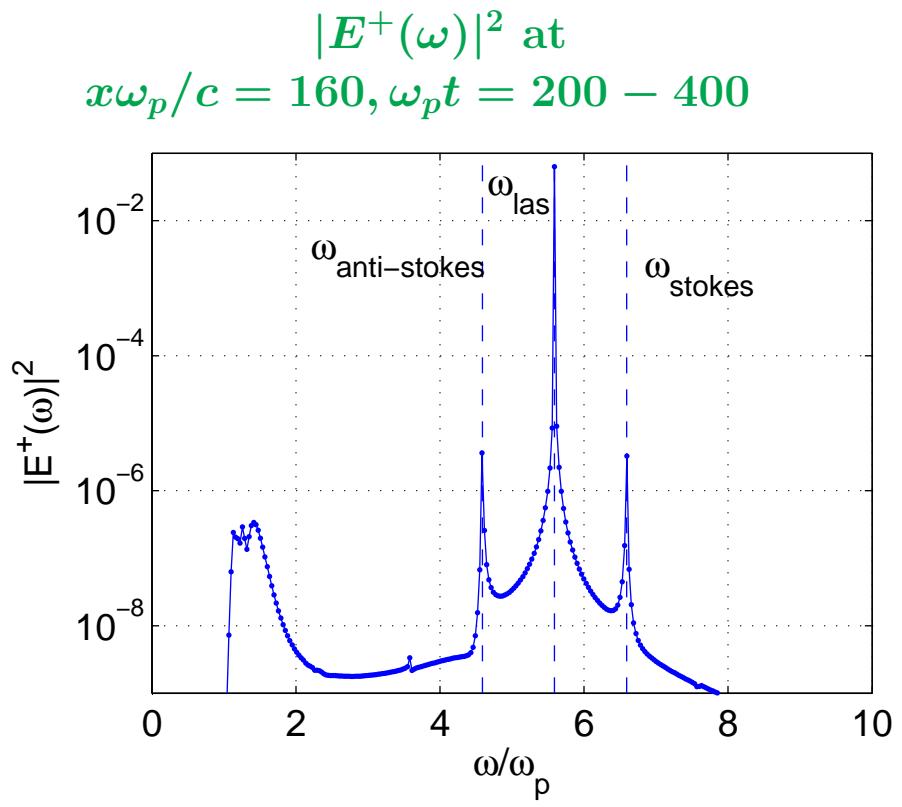


$|n_e(\omega)|^2$  at  $\omega_p t = 450, x\omega_p/c = 30 - 120$



$|n_e(\omega)|^2$  at  $\omega_p t = 1000, x\omega_p/c = 30 - 120$

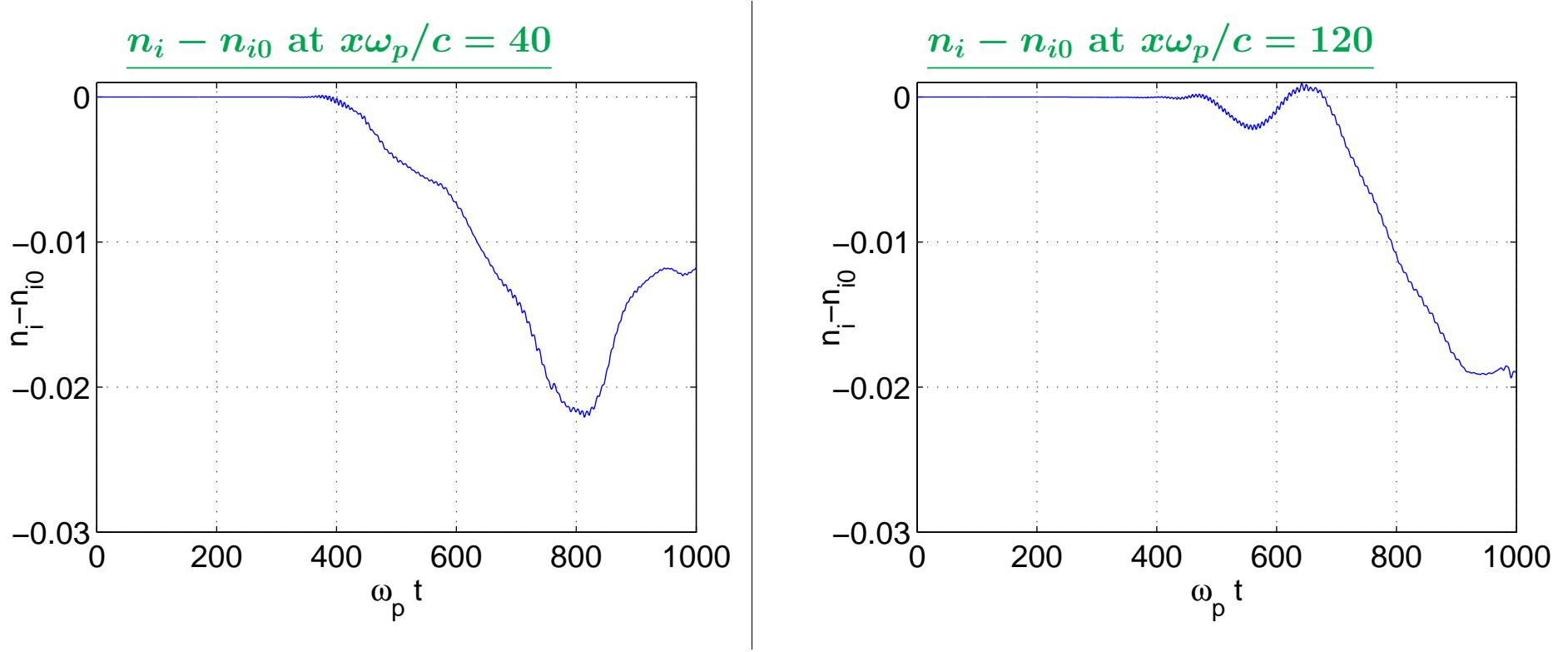
# Transmitted Light $E^+$ at Right Boundary



Forward Raman scattering, both Stokes and anti-Stokes, are present and grow over time

# Ion Density Fluctuations vs. $t$

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Run too short for ion acoustic wave to go through several cycles, e.g.:

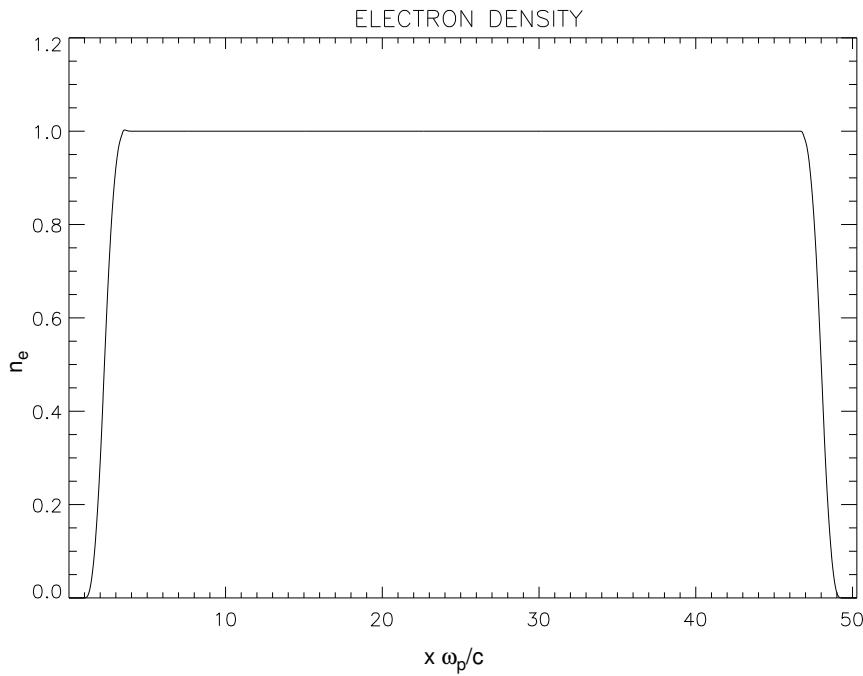
$$c_a = 8.3 \cdot 10^{-4} \quad k_{IAW-SBS} = 11 \quad \rightarrow \quad \tau = \frac{2\pi}{c_a k_{SBS}} = 688$$

# Future Work: Parallelize, $1\frac{1}{2}$ -D Code

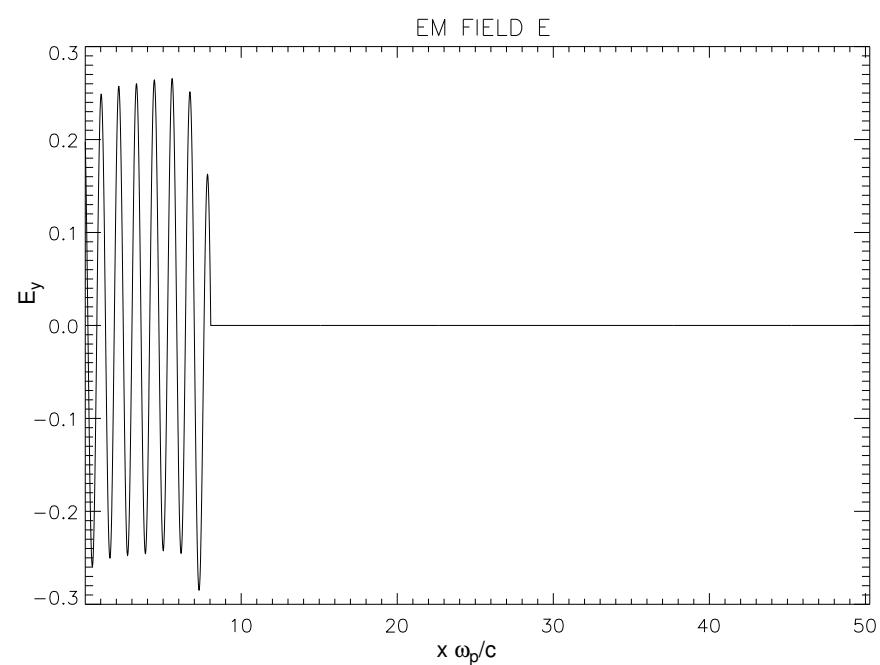
Preliminary results from  $1\frac{1}{2}$ -D code that treats  $v_y \ll c$  kinetically:  $f_s(x, p_x, v_y, t)$ .

$$\frac{\partial f_s}{\partial t} + v_x \frac{\partial f_s}{\partial x} + q_s(E_x + v_y B_z) \frac{\partial f_s}{\partial p_x} + \frac{q_s}{\gamma m_s}(E_y - v_x B_z) \frac{\partial f_s}{\partial v_y} = 0$$

$n_e$  at  $\omega_p t = 8$



$E_y$  at  $\omega_p t = 8$



Comments? Reprint? Please leave address.

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