Dispersion relation for SRS-driven plasma waves with adiabatic electron response

David J. Strozzi\textsuperscript{1}, Didier Bénisti\textsuperscript{2}, Laurent Gremillet\textsuperscript{2}

\textsuperscript{1}A/X Division, LLNL, USA
\textsuperscript{2}CEA/DIF/DPTA, Bruyères-le-Châtel, France

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We model the nonlinear frequency shift of SRS-driven plasma-waves*

- Plasma-wave dispersion relation:  
  \[ 1 + \alpha_d \text{Re}[\chi] = 0 \]

- \( \alpha_d \): ponderomotive drive from light waves:
  - \( \alpha_d = 1 \) for free waves (undriven)\(^2,3\).
  - \( \alpha_d \) related to scattered-wave equation; self-consistent theory particular to SRS.
  - \( \alpha_d \to 1 \) as SRS grows (wave becomes nearly resonant).

- \( \text{Re}[\chi] \): electron susceptibility from nonlinear, adiabatic calculation\(^1\).
  - *Both \( \alpha_d \) and \( \text{Re}[\chi] \) change nonlinearly and reduce plasma-wave frequency.*

- We observe, in Eulerian kinetic simulations, inflation\(^4\) for \( k_p \lambda_D \) up to 0.58:
  - Our dispersion relation agrees with measured plasma-wave frequency.
  - Inflation occurs even for \( k_p \lambda_D > 0.53 \) “loss of resonance” cutoff\(^5\).
  - Because our theory includes amplitude dependence of phase velocity, it does not have a loss-of-resonance cutoff.

  *This work submitted to Phys. Plasmas (Letters).*

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Kinetic simulations of SRS quantify the plasma-wave frequency shift accompanying inflation

\[ \lambda_0 = 351 \text{ nm}, \quad T_e = 5 \text{ keV}, \quad n_e/n_{cr} = 0.1, \quad I_0 = 2 \text{ PW/cm}^2 \implies k_p \lambda_D = 0.448 \]

<table>
<thead>
<tr>
<th>SRS reflectivity</th>
<th>Plasma-wave frequency*</th>
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<tbody>
<tr>
<td><img src="image1" alt="Graph of SRS reflectivity" /></td>
<td><img src="image2" alt="Graph of plasma-wave frequency" /></td>
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</table>

**SRS reflectivity**
- **Run average inflation**
- **Scattered seed**

**Plasma-wave frequency**
- **Linear most unstable mode**
- **Simulation**

*Computed via Hilbert transform of electric field at one \( x \).*

**Measured frequency downshift comparable to undamped growth rate \( \gamma_0 \)**

\[ \gamma_0 \]

**and linear Landau damping rate \( \nu_{p,\text{lin}} \).**

\[ \nu_{p,\text{lin}} \]

\[ \therefore \text{Cannot ignore frequency shift in envelope equations.} \]

Plasma-wave dispersion relation including ponderomotive drive

3-wave model: \( \vec{E} = E_p \sin(\varphi_p) \hat{x} + [E_l \sin \varphi_l + E_s \cos \varphi_s] \hat{y} \)

Nonlinear Raman dispersion relation

\[
1 + \alpha_d \text{Re} \chi = 0 \quad \rightarrow \quad \omega_p
\]

\[\alpha_d \equiv \frac{1 + 2 \eta^{-1} \sin(\delta \varphi) + \eta^{-2}}{1 + \eta^{-1} \sin(\delta \varphi)}\]

\[\delta \varphi \equiv \varphi_p + \varphi_s - \varphi_l\]

\[E_d \equiv \frac{e k_p}{2m \omega_l \omega_s} E_l E_s\]

\[\eta \equiv \frac{E_p}{E_d} \sim \frac{\omega_p}{\nu_{p, \text{Landau}}}\]

Simple linear analog

\[
\partial_t f + v \partial_x f - \frac{e}{m} (E_x + [\vec{v} \times \vec{B}]_x) \partial_v f = 0
\]

Ponderomotive force from light waves

Fourier, linearize \( (1 + \chi) E_p = -\chi E_d \)

\[
(1 + \alpha_d \chi) E_p = 0
\]

\[\alpha_d = 1 + \frac{E_d}{E_p}\]

\[E_d = [\vec{v} \times \vec{B}]_{x, \text{res}}\]

Inflation:

As wave grows, trapping reduces Landau damping:

\[\nu_{p, \text{Land}} \quad \searrow \quad E_d/E_p \quad \searrow \quad \alpha_d \rightarrow 1\]

\(\delta \phi\) and \(\eta\) found self-consistently from scattered wave envelope equation.

D. J. Strozzi, APS-DPP 2007 p. 4
Adiabatic calculation of $\text{Re}[\chi]$, and prior electrostatic theories

• $\text{Re}[\chi]$ for growing wave from adiabatic approximation\(^1\):

\[ \text{valid for slowly-varying fields: } \gamma \equiv \frac{1}{E_p} \frac{\partial E_p}{\partial t} < 0.05 \omega_{pe} \]

• Based on weak action conservation even for separatrix-crossing orbits\(^2\).

• Accounts for amplitude dependence of phase velocity.

• No limitation on $k_p \lambda_D$ or $\omega_{bounce}$.

• Electrostatic theories with constant phase velocity:

  • Dewar\(^3\): adiabatically-excited wave, perturbative:

    \[ \delta \omega_D = \frac{1.09 f''_0(v_p) \omega_{lin}}{1 + (k_p \lambda_D)^2 - \frac{\omega_{lin}^2}{\omega_{pe}^2}} \left[ \frac{e \phi}{T_e} \right]^{1/2} \]

    strictly valid for $k_p \lambda_D \leq 0.3$

  • Morales\(^4\): suddenly-excited wave, perturbative: Dewar 1.09 $\to$ 1.63.

  • Rose\(^5\): BGK steady-state, non-perturbative.

• Bounce-kinetic model\(^6\): driven plasma waves; bounce motion = fast time scale: $\omega_{bounce} \gg \gamma$.

Plasma-wave frequency drops due to both ponderomotive drive ($\alpha_d \to 1$) and nonlinearity in $\text{Re}[\chi]$.

\[ \omega_{srs} : \quad 1 + \alpha_d \text{Re}\chi = 0 \]
\[ \omega_{free} : \quad 1 + \text{Re}\chi = 0 \]

- $\alpha_d$ = SRS ponderomotive drive
- $\text{Re}[\chi]$ from adiabatic theory (Bénisti PoP ‘07)

\[ \delta\omega_D = \frac{1.09 f''(\nu_p)\omega_{lin}}{1 + (k_p\lambda_D)^2 - \frac{\omega_{lin}^2}{\omega_{pe}^2}} \left[ \frac{e\phi}{T_e} \right]^{1/2} \]

- Dewar “adiabatic,” electrostatic calculation

Table:

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<th>Plasma-wave frequency</th>
<th>$\alpha_d \to 1$: plasma wave near resonance</th>
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<td>$\omega_{srs}$</td>
<td>$\omega_{free}$</td>
</tr>
<tr>
<td>$\delta\omega_D$</td>
<td>$\delta\omega_{p,\text{lin}}$</td>
</tr>
<tr>
<td>$\alpha_d$ drops</td>
<td>$e\phi / T_e$</td>
</tr>
<tr>
<td>$\text{Re}[\chi]$ nonlinear</td>
<td>$e\phi / T_e$</td>
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Graph:

- $T_e = 5$ keV
- $k_p\lambda_D = 0.448$
- Simulation
- Linear $\alpha_d$
We observe inflation and model the frequency shift over a range of $k_p \lambda_D$, including $k_p \lambda_D > 0.53$ “loss of resonance” cutoff.

\[ \lambda_0 = 351 \text{ nm, } n_e/n_{cr} = 0.1 \]

Frequency shift at \( e\phi/T_e = 0.1 \)

\[ k_p \lambda_D = 0.53: \text{“loss of resonance” cutoff for } \phi = 0: \text{ no linear solution of } 1 + \text{Re}[\chi] = 0 \]

Undriven theories good for $k_p \lambda_D < 0.35$

\[ \delta \omega_D \text{ (Dewar)} \]
\[ \delta \omega_{\text{free}} (\alpha_d = 1) \]
SRS Inflation observed in simulations above “loss of resonance” cutoff; adiabatic theory matches frequency well

\[ \lambda_0 = 351 \text{ nm}, \ T_e = 9 \text{ keV}, \ n_e/n_{cr} = 0.1 \rightarrow \mathbf{k}_p\lambda_D = 0.578 \] “high”

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<th>SRS reflectivity</th>
<th>Frequency shift</th>
<th>Ponderomotive drive ( \alpha_d )</th>
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<td><img src="image" alt="SRS reflectivity graph" /></td>
<td><img src="image" alt="Frequency shift graph" /></td>
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- SRS drives coherent, large amplitude plasma wave above \( k_p\lambda_D = 0.53 \) “loss-of-resonance” cutoff.
- Our dispersion relation has no loss-of-resonance cutoff, due to amplitude-dependent phase velocity.

But, speckle sideloss and collisions raise the inflation intensity threshold.

D. J. Strozzi, APS-DPP 2007 p. 8
Krook relaxation raises the threshold intensity for inflation

Krook relaxation (mimics speckle sideloss or collisions):

\[
\frac{df}{dt}\bigg|_{\text{Vlasov}} = \nu_K \cdot (f_0 - f)
\]

\[\lambda_0 = 351 \text{ nm}, \quad T_e = 9 \text{ keV}, \quad n_e/n_{cr} = 0.1, \quad \rightarrow \quad k_p \lambda_D = 0.578 \quad \text{“high”}
\]

\[\nu_K = \frac{v_{Te}}{L_\perp}; \quad L_\perp = F \lambda_0
\]

NIF: \( F = 8 \) \( \lambda_0 = 351 \text{ nm}, \quad \nu_K = 8.4 \times 10^{-3} \omega_{pe} \)

But this run was for 75 \( \mu \text{m} \) length, not a full speckle.
Conclusions

• Our dispersion relation for SRS-driven plasma waves:

\[ 1 + \alpha_d \text{Re}[\chi] = 0 \]

- Low amplitude: \( \alpha_d \to 1 \); wave nearly resonant; gives frequency downshift.
- High amplitude: Nonlinearity in \( \text{Re}[\chi] \) gives frequency downshift.

• Electrostatic theories (no ponderomotive drive, \( \alpha_d = 1 \)) under-estimate plasma-wave frequency shift for \( k_p \lambda_D > 0.35 \).

• Inflation observed for \( k_p \lambda_D \) up to 0.58 (no “loss of resonance” cutoff).
  - Our dispersion relation agrees with frequency shift measured in kinetic simulations.
  - Amplitude-dependent phase velocity in \( \text{Re}[\chi] \) relaxes loss-of-resonance cutoff.
  - Sideloss (modeled by Krook relaxation) significantly raises the inflation threshold.

Submitted to Phys. Plasmas (Letters).
Our model: scattered and plasma wave self-consistently coupled

\[ \vec{E} = E_p \sin(\varphi_p)\hat{x} + [E_l \sin \varphi_l + E_s \cos \varphi_s] \hat{y} \]

Raman-driven dispersion relation

\[ 1 + \alpha_d \text{Re} \chi = 0 \Rightarrow \omega_p \]

ponderomotive drive

\[
\begin{align*}
\alpha_d & \equiv \frac{1 + 2\eta^{-1} \sin(\delta \varphi) + \eta^{-2}}{1 + \eta^{-1} \sin(\delta \varphi)} \\
\delta \varphi & \equiv \varphi_p + \varphi_s - \varphi_l \\
E_d & \equiv \frac{ek_p}{2m\omega_l\omega_s} E_l E_s \\
\eta & \equiv \frac{E_p}{E_d} = \frac{\text{Re} \chi}{\text{Im} \chi} \cos \delta \varphi - \sin \delta \varphi \\
\sim & \frac{\omega_p}{\nu_{p,\text{Landau}}} \\
\end{align*}
\]

Scattered light wave envelope equation: from Maxwell equations

\[ [\partial_t + v_{gs} \partial_x + i\Delta_s^{res}] E_s = \Gamma_0 E_p e^{i\delta \varphi} \]

\[ \Gamma_0 \equiv \frac{ek_p}{4m\omega_l\omega_s} E_l \]

\[ (\gamma_s + i\Delta_s^{res}) E_s = \eta \Gamma_0^2 e^{i\delta \varphi} \]

\[ \gamma_s = E_s^{-1} [\partial_t E_s + v_{gs} \partial_x E_s] \]

\[ \Delta_s^{res} \equiv \frac{\omega_s^2 - k_s c^2 - \omega_p^2}{2\omega_s} \]

Nonlinear enhancement of growth rate and detuning of scattered wave both important!
Ponderomotive drive in the plasma wave dispersion relation increases its frequency compared to a free wave.

Driven dispersion relation: \[ 1 + \alpha_d \text{Re} \chi = 0; \quad \alpha_d > 1 \]

Free dispersion relation: \[ 1 + \text{Re} \chi = 0; \quad \alpha_d \rightarrow 1 \quad \text{“resonance”} \]

Linear theory:

\[ Z'_r (\zeta) = 2 \bar{K}^2 \]

\[ \bar{K} \equiv \frac{k_p \lambda_D}{\sqrt{\alpha_d}} \quad \zeta \equiv \frac{\omega_p}{k_p v_T \sqrt{2}} \]

\[ k_p \lambda_D = 0.448 \]

Inflation:

As the wave grows, trapping reduces Landau damping:

\[ \text{Im}[\chi] \Downarrow \quad \eta \uparrow \quad \alpha_d \rightarrow 1 \quad \omega_p \rightarrow \omega_{\text{free}} \]

\[ \alpha_d \equiv \frac{1 + 2\eta^{-1} \sin(\delta \varphi) + \eta^{-2}}{1 + \eta^{-1} \sin(\delta \varphi)} \]

\[ \eta \equiv \frac{E_p}{E_d} = \frac{\text{Re} \chi}{\text{Im} \chi} \cos \delta \varphi - \sin \delta \varphi \]

D. J. Strozzi, APS-DPP 2007 p. 12
“Loss of resonance” found in theories that neglect amplitude dependence of phase velocity

Rose dispersion relation\(^1\):
\[ \varepsilon_r = 1 + \text{Re}[\chi] = 0 \]

Landau \(\varepsilon = 0\)

\(\varepsilon_r \neq 1 + \text{Re}[\chi] = 0\)

\(\varepsilon_r |_{min.} \)

Inflation:
Trapping flattens \(f\), reduces \(\varepsilon_i\)
\(\Rightarrow\) gain increases