Dispersion relation for SRS-driven plasma waves with adiabatic electron response

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We model the nonlinear frequency shift of SRS-driven plasma-waves*

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- Plasma-wave dispersion relation: $1+\alpha_d \mathrm{Re}[\chi]=0$ (↑ α_d: pond. drive)

*This work submitted to Phys. Plasmas (Letters).

- α_d: ponderomotive drive from light waves:
  - $\alpha_d=1$ for free waves (undriven)[2,3].
  - α_d related to scattered-wave equation; self-consistent theory particular to SRS.
  - $\alpha_d \rightarrow 1$ as SRS grows (wave becomes nearly resonant).

- **Re[χ]:** electron susceptibility from nonlinear, adiabatic calculation[1].
- *Both* α_d *and* Re[χ] change nonlinearly and reduce plasma-wave frequency.

- We observe, in Eulerian kinetic simulations, inflation[4] for $k_p\lambda_D$ up to 0.58:
  - Our dispersion relation agrees with measured plasma-wave frequency.
  - Inflation occurs even for $k_p\lambda_D>0.53$ "loss of resonance" cutoff[5].
  - Because our theory includes amplitude dependence of phase velocity, it does not have a loss-of-resonance cutoff.

Frequency shift an ingredient in envelope equations that capture kinetic effects.

[1] D. Bénisti and L. Gremillet, Phys. Plasmas **14**, 042304 (2007).
[2] R. L. Dewar, Phys. Fluids **15**, 712 (1972).
[3] G. J. Morales and T. M. O'Neil, Phys. Rev. Lett. **28**, 417 (1972).
[4] H. X. Vu, D. F. DuBois, and B. Bezzerides, Phys. Rev. Lett. **86**, 4306 (2001).
[5] H. A. Rose and D. A. Russell, Phys. Plasmas **8**, 4784 (2001).

Kinetic simulations of SRS quantify the plasma-wave frequency shift accompanying inflation

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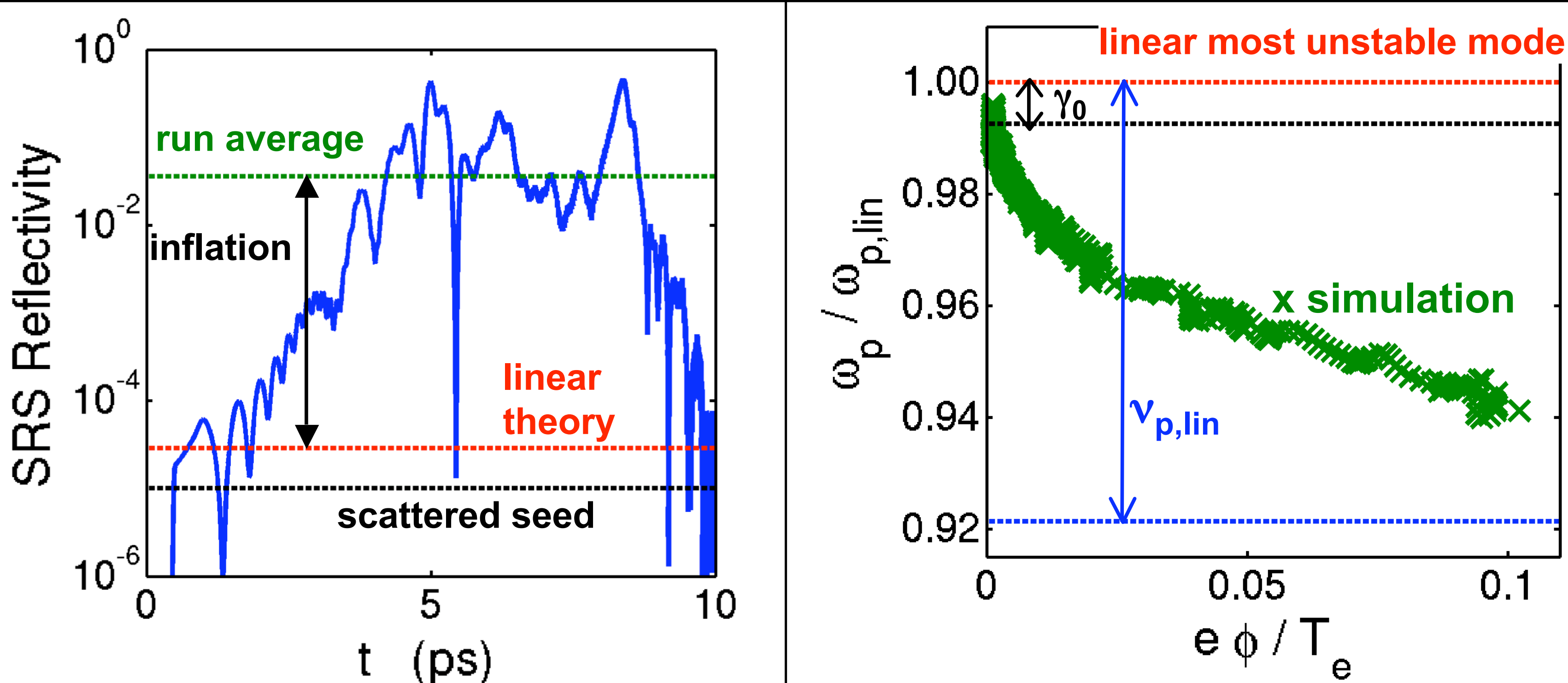
λ_0=351 nm, T_e=5 keV, n_e/n_{cr}=0.1, I_0=2 PW/cm^2 → $k_p\lambda_D$=0.448

SRS reflectivity	Plasma-wave frequency*



*Computed via Hilbert transform of electric field at one x.

Measured frequency downshift comparable to undamped growth rate γ_0 and linear Landau damping rate $\nu_{p,lin}$.

∴ Cannot ignore frequency shift in envelope equations.

Simulations: Vlasov-Maxwell solver ELVIS [D. J. Strozzi et al., Phys. Plasmas 14, 013104 (2007)]

Plasma-wave dispersion relation including ponderomotive drive

3-wave model: $\vec{E} = E_p \sin(\varphi_p)\hat{x} + [E_l \sin\varphi_l + E_s \cos\varphi_s]\hat{y}$

Nonlinear Raman dispersion relation

ponderomotive drive

$$1 + \alpha_d \mathrm{Re}\chi = 0 \quad \rightarrow \quad \omega_p$$

$$\alpha_d \equiv \frac{1 + 2\eta^{-1}\sin(\delta\varphi) + \eta^{-2}}{1 + \eta^{-1}\sin(\delta\varphi)}$$

$$\delta\varphi \equiv \varphi_p + \varphi_s - \varphi_l$$ **phase mismatch**

$$E_d \equiv \frac{ek_p}{2m\omega_l\omega_s} E_l E_s$$ **pond. drive**

$$\eta \equiv \frac{E_p}{E_d} \sim \frac{\omega_p}{\nu_{\mathrm{p,Landau}}}$$ **plasma response / pond. drive**

$\delta\phi$ and η found self-consistently from scattered wave envelope equation.

Simple linear analog

$$\partial_t f + v\partial_x f - \frac{e}{m}(E_x + [\vec{v} \times \vec{B}]_x)\partial_v f = 0$$

ponderomotive force from light waves

Fourier, linearize $\longrightarrow$ $(1+\chi)E_p = -\chi E_d$

$$\longrightarrow \quad (1 + \alpha_d \chi)E_p = 0$$

$$\alpha_d = 1 + \frac{E_d}{E_p}$$

$$E_d = [\vec{v} \times \vec{B}]_{x,res}$$

Inflation:

As wave grows, trapping reduces Landau damping:

$\nu_{p,Land.}\searrow$ ➢ $E_d/E_p \searrow$ ➢ $\alpha_d \rightarrow 1$

Adiabatic calculation of Re[χ], and prior electrostatic theories

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$$1+\alpha_d \mathrm{Re}[\chi]=0$$

- Re[χ] for growing wave from adiabatic approximation[1]:
  - Valid for slowly-varying fields: $\gamma \equiv \frac{1}{E_p}\frac{\partial E_p}{\partial t} < 0.05\omega_{pe}$
  - Based on weak action conservation even for separatrix-crossing orbits[2].
  - Accounts for amplitude dependence of phase velocity.
  - No limitation on $k_p\lambda_D$ or ω_{bounce}.

$$1+\mathrm{Re}[\chi]=0; \quad \alpha_d \rightarrow 1$$

- Electrostatic theories with constant phase velocity:
  - Dewar[3]: adiabatically-excited wave, perturbative:

$$\delta\omega_D = \frac{1.09 f_0''(v_p)\omega_{lin}}{1+(k_p\lambda_D)^2 - \frac{\omega_{lin}^2}{\omega_{pe}^2}}\left[\frac{e\phi}{T_e}\right]^{1/2}$$

strictly valid for $k_p\lambda_D \leq 0.3$

  - Morales[4]: suddenly-excited wave, perturbative: Dewar 1.09 → 1.63.
  - Rose[5]: BGK steady-state, non-perturbative.
- Bounce-kinetic model[6]: driven plasma waves; bounce motion = fast time scale: $\omega_{bounce} >> \gamma$.

[1] D. Bénisti, L. Gremillet, Phys. Plasmas **14**, 042304 (2007).
[2] J. R. Cary, D. F. Escande, J. L.. Tennyson, Phys. Rev. A **34**, 4256 (1986).
[3] R. L. Dewar, Phys. Fluids **15**, 712 (1972); [4] G. J. Morales and T. M. O'Neil, Phys. Rev. Lett. **28**, 417 (1972).
[5] H. A. Rose and D. A. Russell, Phys. Plasmas **8**, 4784 (2001); [6] D. C. Barnes, Phys. Plasmas **11**, 903 (2004).

Plasma-wave frequency drops due to both ponderomotive drive ($\alpha_d \rightarrow 1$) and nonlinearity in Re[χ]

$$\omega_{srs}: \quad 1 + \alpha_d \mathrm{Re}\chi = 0$$

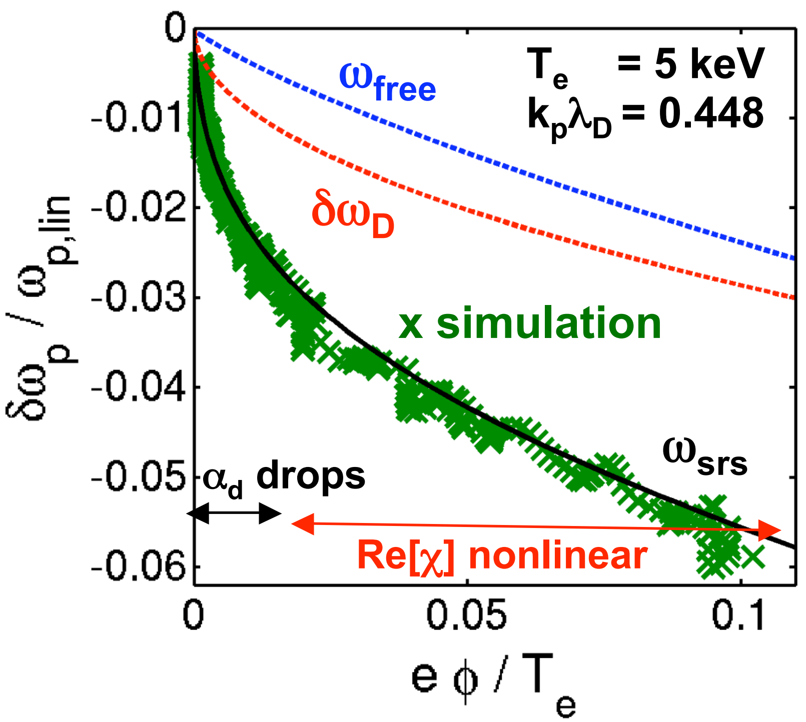
$$\omega_{free}: \quad 1 + \mathrm{Re}\chi = 0$$

- α_d = SRS ponderomotive drive
- Re[χ] from adiabatic theory (Bénisti PoP '07)

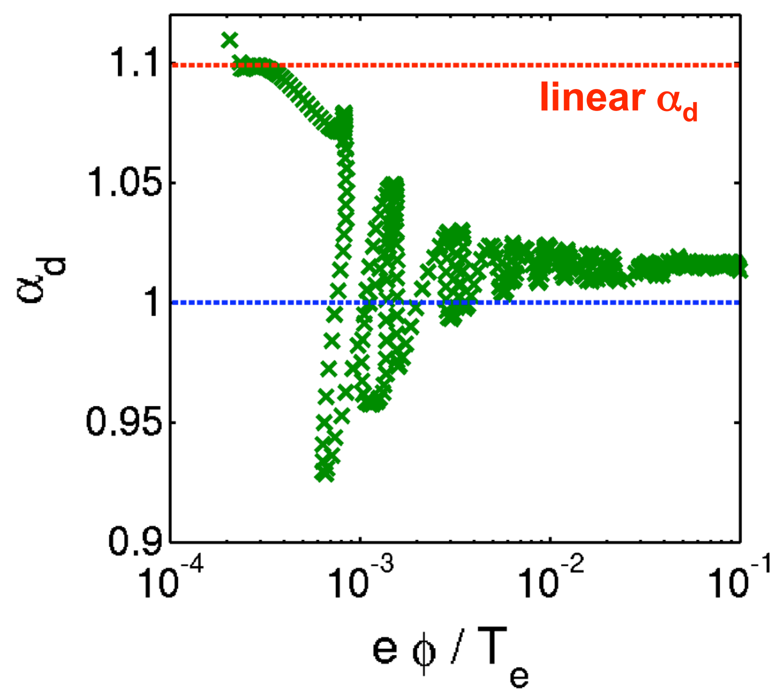
$$\delta\omega_D = \frac{1.09 f_0''(v_p)\omega_{lin}}{1 + (k_p\lambda_D)^2 - \frac{\omega_{lin}^2}{\omega_{pe}^2}} \left[\frac{e\phi}{T_e}\right]^{1/2}$$

- Dewar "adiabatic," electrostatic calculation

Plasma-wave frequency



$\alpha_d \rightarrow 1$: plasma wave near resonance



We observe inflation and model the frequency shift over a range of $k_p\lambda_D$, including $k_p\lambda_D$>0.53 "loss of resonance" cutoff

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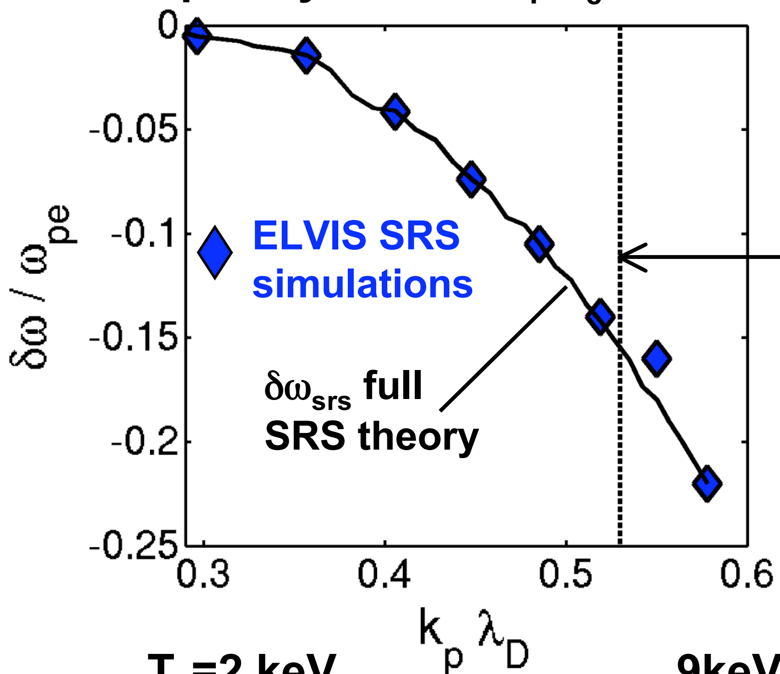
λ_0=351 nm, n_e/n_{cr}=0.1

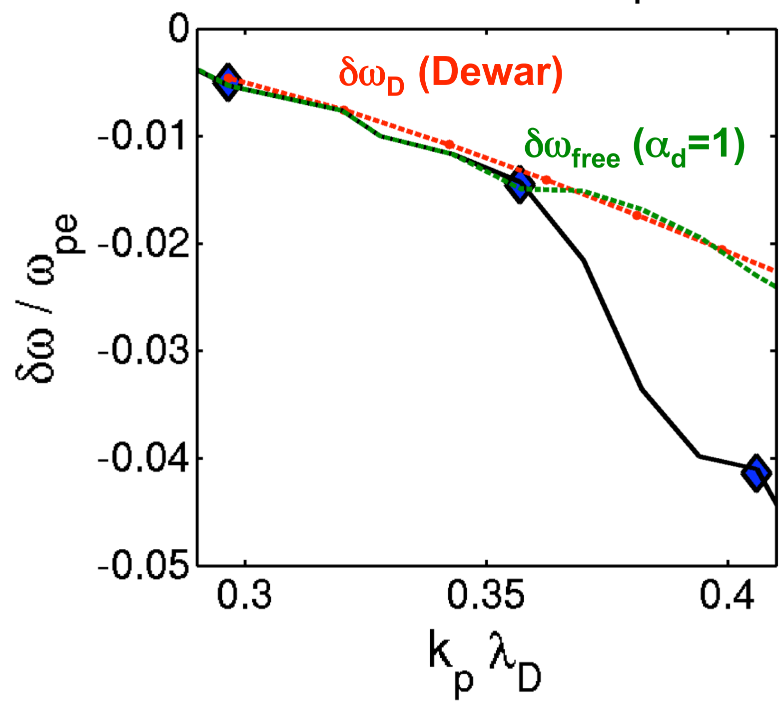
Frequency shift at $e\phi/T_e$ = 0.1



T_e=2 keV **9keV**

$k_p\lambda_D$=0.53: "loss of resonance" cutoff for ϕ=0: no linear solution of 1+Re[χ]=0

Undriven theories good for $k_p\lambda_D$<0.35



SRS Inflation observed in simulations above "loss of resonance" cutoff; adiabatic theory matches frequency well

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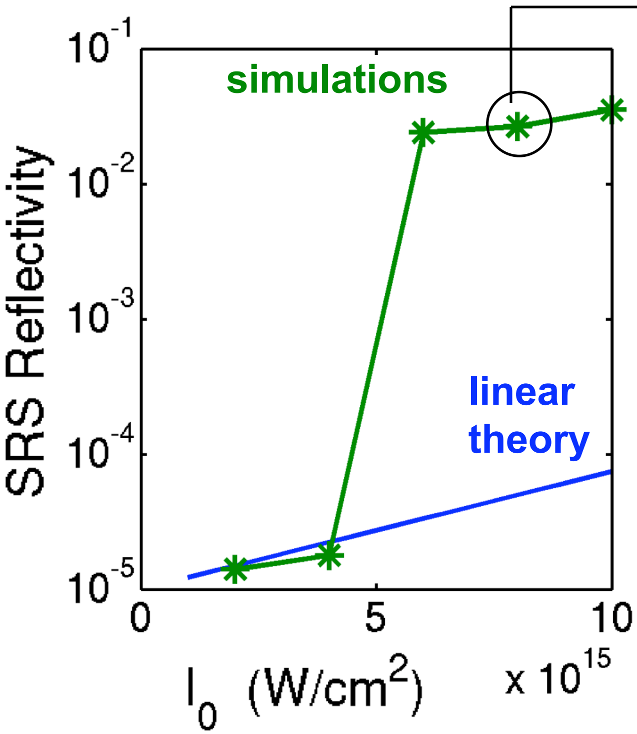
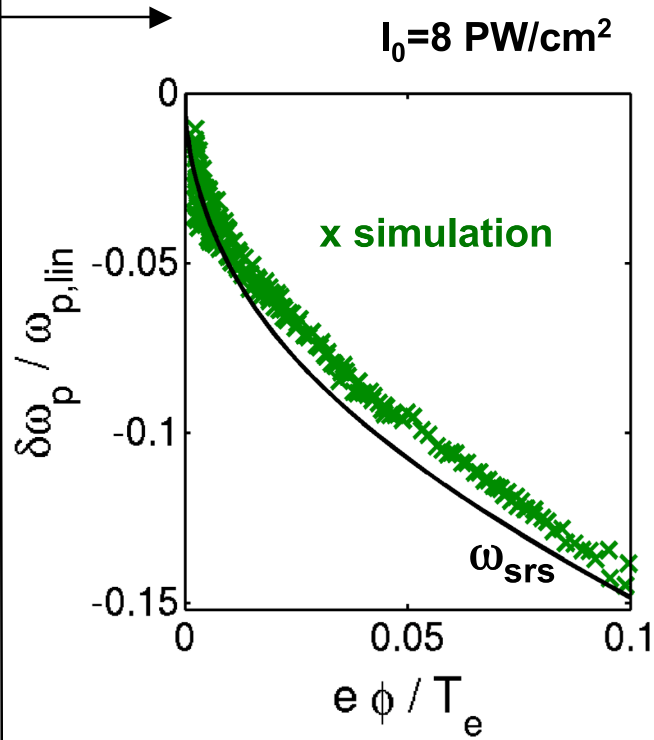
λ_0=351 nm, T_e=9 keV, n_e/n_{cr}=0.1 → **$k_p\lambda_D$=0.578** **"high"**

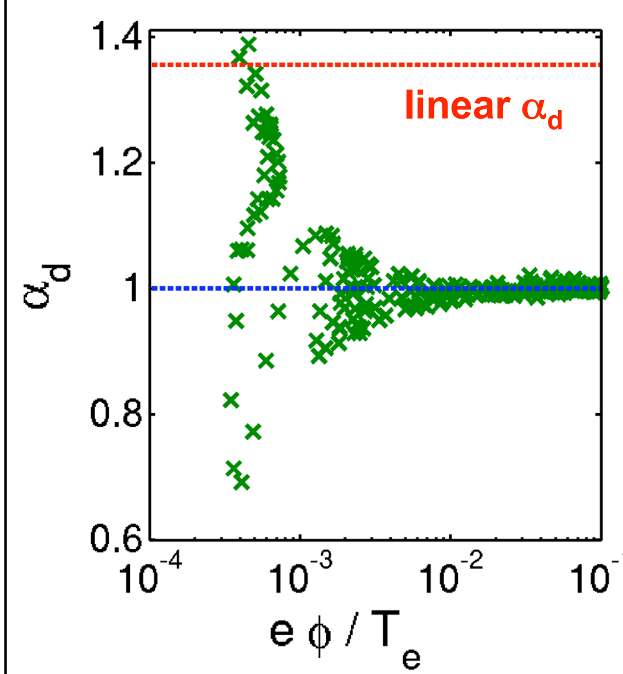
SRS reflectivity



Frequency shift



Ponderomotive drive α_d



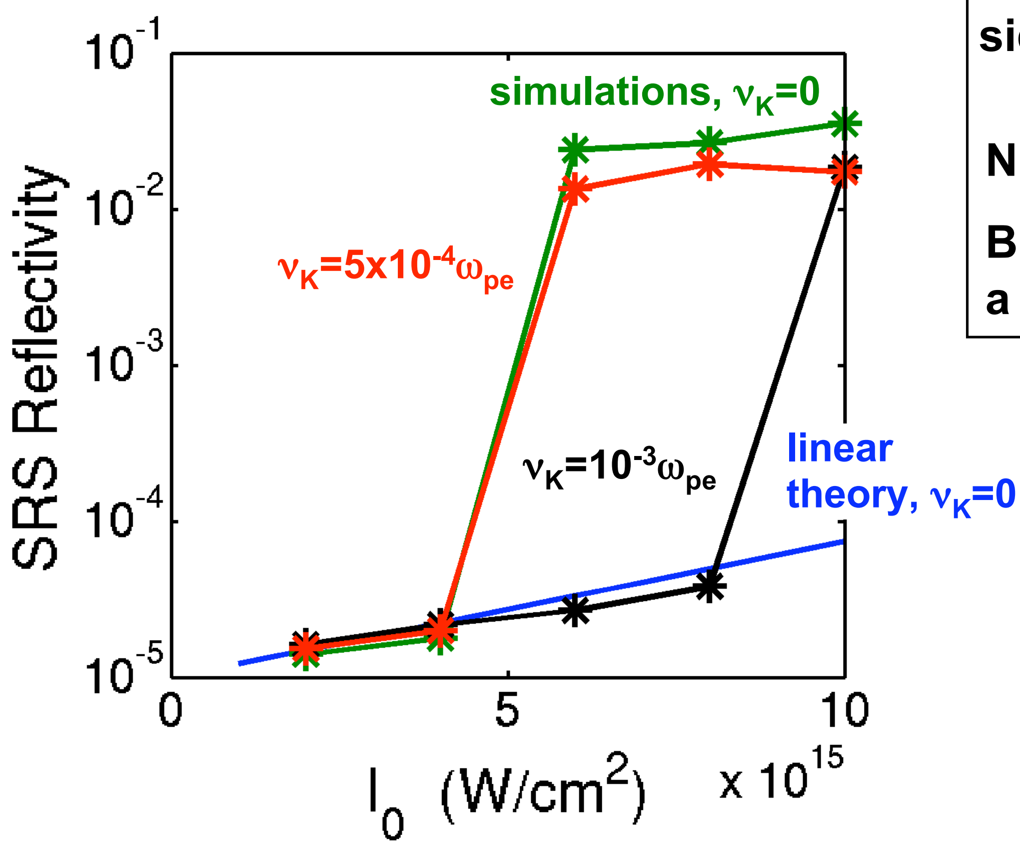
- SRS drives coherent, large amplitude plasma wave above $k_p\lambda_D$=0.53 "loss-of-resonance" cutoff.
- Our dispersion relation has no loss-of-resonance cutoff, due to amplitude-dependent phase velocity.

But, speckle sideloss and collisions raise the inflation intensity threshold.

Krook relaxation raises the threshold intensity for inflation

Krook relaxation (mimics speckle sideloss or collisions): $\left.\frac{df}{dt}\right|_{\mathrm{Vlasov}} = \nu_K \cdot (f_0 - f)$

λ_0=351 nm, T_e=9 keV, n_e/n_{cr}=0.1, → **$k_p\lambda_D$=0.578** **"high"**



sideloss: $\nu_K = \frac{v_{Te}}{L_\perp}; \qquad L_\perp = F\lambda_0$

NIF: F=8 λ_0=351 nm, ν_K=8.4x10^{-3} ω_{pe}

But this run was for 75 μm length, not a full speckle.

Conclusions

- Our dispersion relation for SRS-driven plasma waves:

Submitted to Phys. Plasmas (Letters).

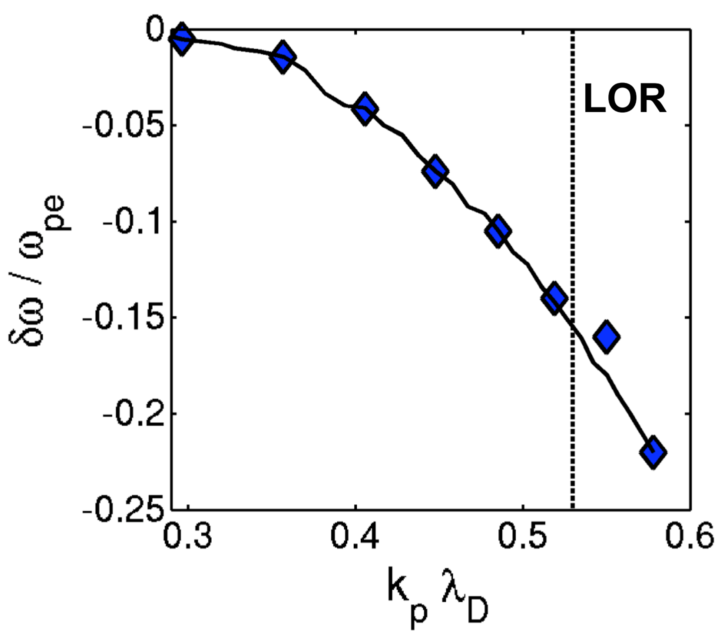
$$1+\alpha_d \mathrm{Re}[\chi]=0$$

pond. drive

- Low amplitude: $\alpha_d \to 1$; wave nearly resonant; gives frequency downshift.
- High amplitude: Nonlinearity in Re[χ] gives frequency downshift.

- Electrostatic theories (no ponderomotive drive, α_d=1) under-estimate plasma-wave frequency shift for $k_p\lambda_D$>0.35.

- Inflation observed for $k_p\lambda_D$ up to 0.58 (no "loss of resonance" cutoff).
  - Our dispersion relation agrees with frequency shift measured in kinetic simulations.
  - Amplitude-dependent phase velocity in Re[χ] relaxes loss-of-resonance cutoff.
  - Sideloss (modeled by Krook relaxation) significantly raises the inflation threshold.



Our model: scattered and plasma wave self-consistently coupled

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$$\vec{E} = E_p \sin(\varphi_p)\hat{x} + [E_l \sin\varphi_l + E_s \cos\varphi_s]\hat{y}$$

Raman-driven dispersion relation

ponderomotive drive

$$1 + \alpha_d \mathrm{Re}\chi = 0 \quad \rightarrow \quad \omega_p$$

$$\alpha_d \equiv \frac{1 + 2\eta^{-1}\sin(\delta\varphi) + \eta^{-2}}{1 + \eta^{-1}\sin(\delta\varphi)}$$

$$\delta\varphi \equiv \varphi_p + \varphi_s - \varphi_l$$

$$E_d \equiv \frac{ek_p}{2m\omega_l\omega_s} E_l E_s$$

$$\eta \equiv \frac{E_p}{E_d} = \frac{\mathrm{Re}\chi}{\mathrm{Im}\chi}\cos\delta\varphi - \sin\delta\varphi$$

$$\sim \frac{\omega_p}{\nu_{\mathrm{p,Landau}}}$$

Scattered light wave envelope equation: from Maxwell equations

$$[\partial_t + v_{gs}\partial_x + i\Delta_s^{res}]\, E_s = \Gamma_0 E_p e^{i\delta\varphi}$$

$$\Gamma_0 \equiv \frac{ek_p}{4m\omega_l\omega_s} E_l$$

$$\rightarrow \quad (\gamma_s + i\Delta_s^{res})E_s = \eta\Gamma_0^2 e^{i\delta\varphi}$$

$$\gamma_s \equiv E_s^{-1}\left[\partial_t E_s + v_{gs}\partial_x E_s\right]$$

$$\Delta_s^{res} \equiv \frac{\omega_s^2 - k_s^2 c^2 - \omega_{pe}^2}{2\omega_s}$$

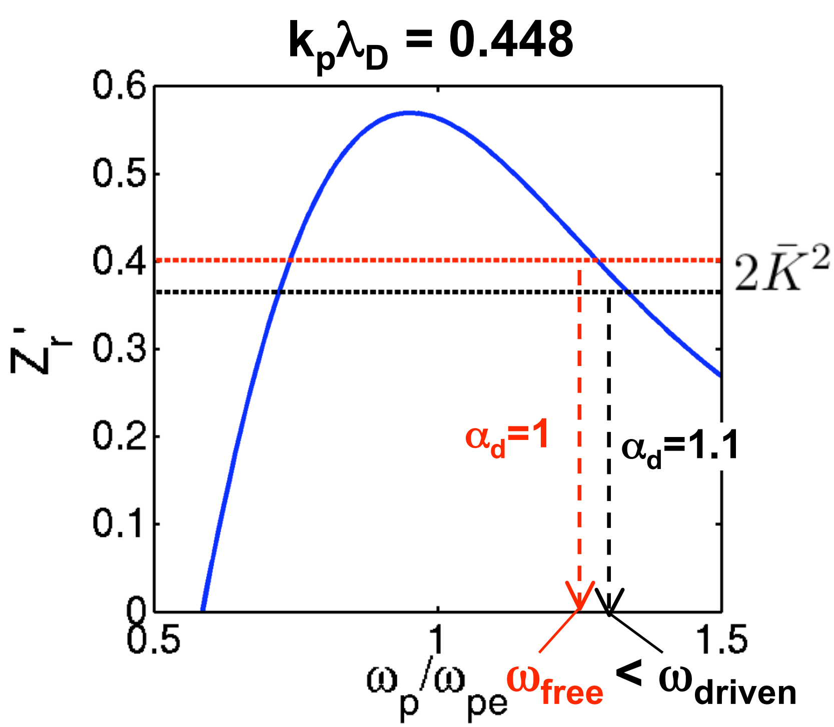
Nonlinear enhancement of growth rate *and* detuning of scattered wave *both* important!

Ponderomotive drive in the plasma wave dispersion relation increases its frequency compared to a free wave

Driven dispersion relation: $1 + \alpha_d \mathrm{Re}\chi = 0; \quad \alpha_d > 1$

Free dispersion relation: $1 + \mathrm{Re}\chi = 0; \quad \alpha_d \to 1$ **"resonance"**

Linear theory: $\boxed{Z_r'(\zeta) = 2\bar{K}^2}$ $\quad \bar{K} \equiv \frac{k_p \lambda_D}{\sqrt{\alpha_d}} \quad \zeta \equiv \frac{\omega_p}{k_p v_{Te}\sqrt{2}}$



Inflation:

As the wave grows, trapping reduces Landau damping:

Im[χ]↘ ➢ η↗ ➢ α_d→1 ➢ ω_p→ω_{free}

$$\alpha_d \equiv \frac{1 + 2\eta^{-1}\sin(\delta\varphi) + \eta^{-2}}{1 + \eta^{-1}\sin(\delta\varphi)}$$

$$\eta \equiv \frac{E_p}{E_d} = \frac{\mathrm{Re}\chi}{\mathrm{Im}\chi}\cos\delta\varphi - \sin\delta\varphi$$

"Loss of resonance" found in theories that neglect amplitude dependence of phase velocity

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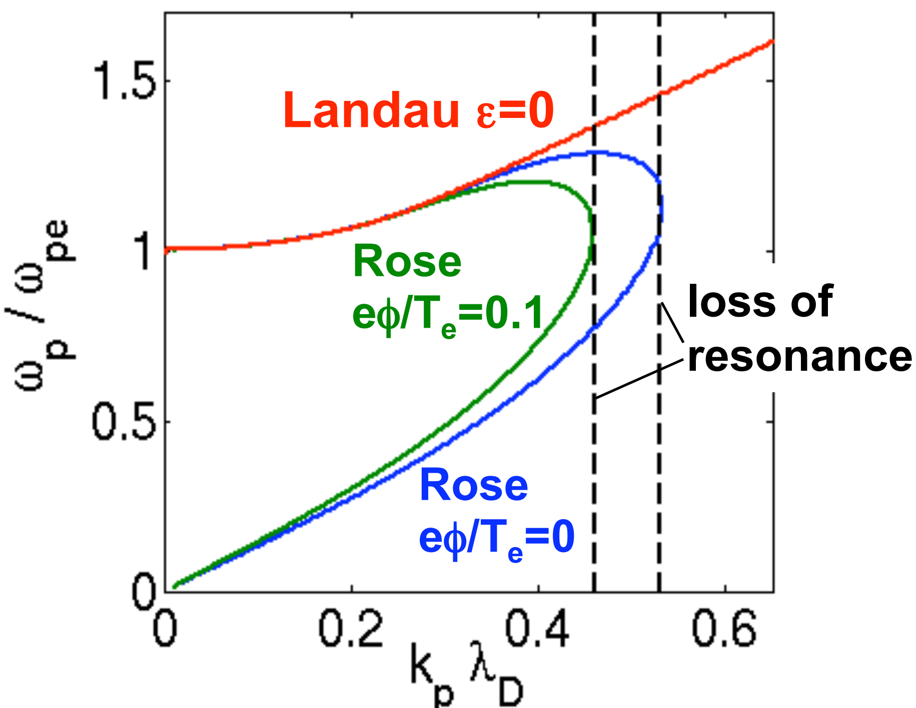
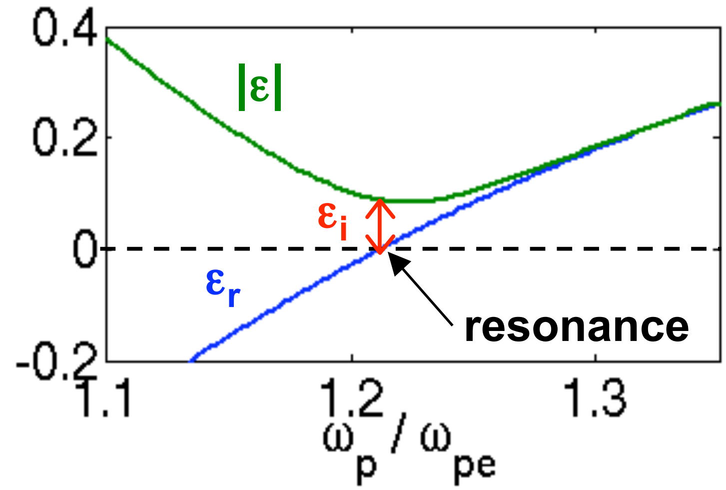
Rose dispersion relation[1]: $\varepsilon_r=1+\mathrm{Re}[\chi]=0$



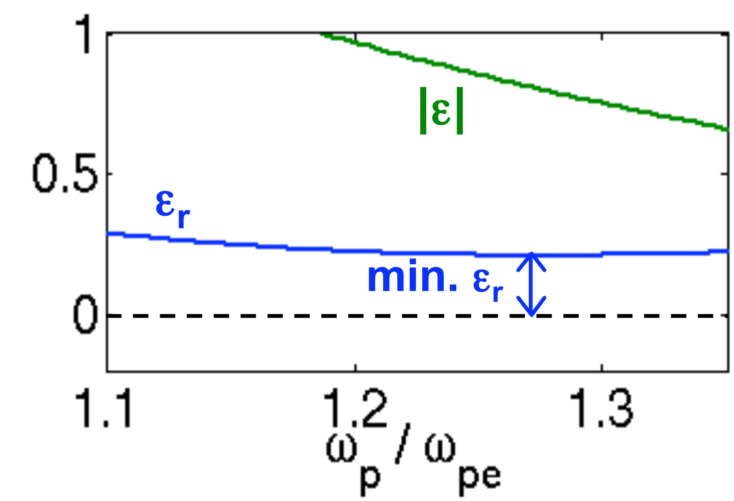
Inflation:
Trapping flattens f,
reduces ε_i
→ gain increases

$$\text{SRS gain} \sim \frac{1}{|\epsilon|^2}$$

$k_p\lambda_D=0.35$: resonance exists: trapping can cause inflation



$k_p\lambda_D=0.6$: loss of resonance: inflation harder due to ε_r



[1] H. A. Rose and D. A. Russell, Phys. Plasmas 8, 4784 (2001).