KINETIC SIMULATION OF LASER-PLASMA INTERACTIONS

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Abstract

We use a one-dimensional Eulerian Vlasov Code to study laser-plasma interactions. The excitation of parametric instabilities, such as stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS), is an important issue for the efficiency of inertial confinement fusion. Our aim is to understand the growth, saturation, and coupling of these instabilities, in particular the kinetic aspects of these phenomena.

The code evolves both species with the fully relativistic Vlasov equation in the direction of laser propagation. It also incorporates a cold fluid transverse velocity for each species, which is needed to couple the transverse and longitudinal dynamics. The plasma is finite and not periodic. On the boundaries we place “absorbing plates” that collect the particles flowing onto them. This charge enters Poisson’s equation as a boundary condition.

We run the code for much longer than it takes the laser to cross the computational domain. Once the laser passes a region of plasma, it modifies the equilibrium. This modification is sufficient for SRS to grow from; we do not “seed” SRS by imposing perturbations or a secondary laser.

For parameters similar to the recent Trident single hot spot experiments, SRS is not limited by the Langmuir Decay Instability (LDI) or other ion processes, but by kinetic effects. Large vortices form in phase-space due to electron trapping in the field of the SRS electron plasma wave. At the observed amplitudes of the electrostatic field, the process of wave-breaking may be relevant. Although we have not yet observed stimulated electron acoustic scattering (SEAS), beam-like structures in the electron distribution develop off of which SEAS may occur.

We also see SBS starting to develop, but we have not run for long enough times for the ions to move through several ion-acoustic wave periods.

Work is underway to extend the code to $1 \frac{1}{2}$ dimensions and to parallelize it.
Single Hot Spot Experiments


**One-Dimensional Hot Spot Model Geometry**
1-Dimensional Kinetic Model

- Spatial variation only in $x$: $\nabla \rightarrow \partial / \partial x$
- Fields: $\vec{E} = (E_x, E_y, 0)$, $\vec{B} = (0, 0, B_z)$
- Transverse cold beam: $F_s(x, p_x, P_y, t) = \delta(P_y) f_s(x, p_x, t)$, $P_y = m_s v_{ys} + q_s A_y = y$ canonical mom. $y$ momentum equation becomes [Gabor, Proc. IRE 33:792, 1945]

\[
\frac{\partial v_{ys}}{\partial t} = \frac{q_s}{m_s} E_y
\]

- Longitudinal dynamics: Relativistic 1-D Vlasov Eqn. for $f_s(x, p_x, t)$

\[
\frac{\partial f_s}{\partial t} + v_x \frac{\partial f_s}{\partial x} + \frac{q_s}{m_s} (E_x + v_{ys} B_z) \frac{\partial f_s}{\partial p_x} = 0
\]

- Longitudinal field: $E_x$ (electrostatic perturbations)

\[
\text{Poisson} \rightarrow \frac{\partial E_x}{\partial x} = \frac{e}{\epsilon_0} (n_i - n_e)
\]

- Transverse fields: laser, electromagnetic perturbations

\[
\text{Ampère} \rightarrow \frac{\partial E_y}{\partial t} + c^2 \frac{\partial B_z}{\partial x} = -\frac{1}{\epsilon_0} J_y
\]

\[E^\pm = E_y \pm cB_z \rightarrow \left( \frac{\partial}{\partial t} \pm c \frac{\partial}{\partial x} \right) E^\pm = -\frac{1}{\epsilon_0} J_y\]
The Time-Stepping Algorithm


\[
\begin{align*}
t/dt & \quad 0 & 0^+ & 1^- & 1^+ & 1^- & 1^+ \\
\text{(leapfrog)} & \quad f & f_0 & (\partial_t + v_x \partial_x) f = 0 & f_{1-} & (\partial_t + v_x \partial_x) f = 0 & f_{1+} & (\partial_t + v_x \partial_x) f = 0 \\
\text{(what we do)} & \quad f & f_0 & (\partial_t + v_x \partial_x) f = 0 & f_{1-} & (\partial_t + v_x \partial_x) f = 0 & f_{1+} & (\partial_t + v_x \partial_x) f = 0 \\
n & \quad n_0 & n_1 & \frac{1}{2} (n_0 + n_1) & n_1[f_{1-}] \\
N_p & \quad N_{p0} & N_{p1} & \frac{1}{2} (N_{p0} + N_{p1}) & N_{p1}[n_1] \\
dN_w & \quad dN_{w0} & dN_{w0} & \frac{1}{2} (dN_{w0} + dN_{w1}) & dN_{w1}[f_{1-}] \\
N_w & \quad N_{w0} & N_{w0} + \frac{1}{2} dN_{w1} & N_{w1} = N_{w0} + dN_{w1} \\
E_x & \quad E_{x \frac{1}{2}} [n_1, N_{p \frac{1}{2}}, N_{w \frac{1}{2}}] \\
E^\pm & \quad E_{0+}^\pm & (\partial_t + v_x) E^\pm = 0 & \frac{1}{2} (E_{0+}^\pm + E_{1-}^\pm) & E_{1-}^\pm & (\partial_t + v_x) E^\pm = -e_0^{-1} J_y^1 & E_{1+}^\pm \\
v_y & \quad v_{y0} & m \partial_t v_y = qE_{\frac{1}{2}} & v_{y1} & \frac{1}{2} (v_{y0} + v_{y1}) & v_{y1}
\end{align*}
\]
Boundary Conditions

- Nonperiodic grids in $x$ and $p_x$; plasma is finite.
- “Absorbing plates” (transparent to electromagnetic fields) at $x = 0$ and $x = L$. Particles that flow out to the plates stay there and no longer evolve.

\[
N_{sL}, \; N_{sP}, \; N_{sR} = \# \text{ of particles of species } s \text{ on left plate, in plasma, on right plate}
\]

\[
X(p) = -dt \; v(p) \quad v(p) = \frac{p}{\gamma m}
\]

\[
\Delta N_L = \int_{p_{min}}^{0} dp \int_{0}^{X(p)} dx f(x, p)
\]

\[
\approx -dt \int_{p_{min}}^{0} dp \; v \; f(0, p)
\]

- Charges on plates enter $E_x$ as a boundary condition.

\[
\frac{\partial E_x}{\partial x} = \epsilon_0^{-1} \rho
\]

\[
Q_L + Q_P + Q_R = 0
\]

\[
\therefore \; E_x(x < \text{left plate}) = 0
\]

\[
E(x = 0) = \frac{1}{2\epsilon_0}(Q_L - Q_P - Q_R)
\]

\[
E(x) = E(x = 0) + \epsilon_0^{-1} \int_{0}^{x} \rho(x')dx'
\]
Outflow to plates

Change in total number of particles

Electrons on left, right walls

Ions on left, right walls
Run Parameters

• **Scales used in code:**

  time: \( t_0 = \omega_p^{-1} \)  
  length: \( d_e = c/\omega_p \)  
  speed: \( c \)  
  momentum: \( p_0 = m_e c \)  
  E field: \( v_{quiv}(E_0x) = c \)

• **Plasma Parameters:**

  Plasma length: \( L = 75 \ \mu m = 160 \ d_e \)  
  Plasma density: \( n_e = 1.28 \cdot 10^{20} \ cm^{-3} = 0.032 \ n_{crit} \)  
  initial electron Temperature: \( T_e = 350 \ eV \)  
  initial ion Temperature: \( T_i = 100 \ eV \)  
  electron thermal speed: \( v_{Te} = 0.0262 \ c \)  
  ion thermal speed: \( v_{Ti} = 3.27 \cdot 10^{-4} \ c \)  
  Plasma frequency: \( \omega_p = 6.4 \cdot 10^{14} \ rad/s \rightarrow t_0 = 1.56 \cdot 10^{-15} \ s, \ d_e = 470 \ nm \)  
  Debye length: \( \lambda_{De} = 12.3 \ nm = 0.0262 \ d_e \)

• **Laser Parameters:**

  Laser frequency: \( \omega_0 = 3.57 \cdot 10^{15} \ rad/s = 5.59 \ \omega_p \)  
  free-space wavelength: \( \lambda_0 = 527 \ nm = 1.12 \ d_e \)  
  Laser instensity: \( I_0 = 10^{16} \ W/cm^2 \)  
  electron quiver velocity: \( v_{quiv} = .045 \ c \)

• **Numerical parameters:**

  Timestep: \( dt = c \ dx = 0.01/\omega_p \)  
  Total runtime: \( T = 1.56 \ ps = 1000 \ t_0 \)  
  Spatial gridpoints: 16001 \( \rightarrow dx = 0.01 \ d_e \)  
  electron momentum gridpoints: 301 \( \rightarrow dp_e = 0.0027 \ m_e c = 0.102 \ p_{Te} \)  
  ion momentum gridpoints: 151 \( \rightarrow dp_i = 0.069 \ m_e c = 0.187 \ p_{Ti} \)
Parametric Instabilities: Coupling of Modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Equation</th>
<th>Units</th>
<th>Polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMW (electromagnetic)</td>
<td>$\omega^2 = \omega_{pe}^2 + c^2k^2$</td>
<td>e/m</td>
<td>$\vec{E} \perp \vec{k}$</td>
</tr>
<tr>
<td>EPW ($e^-$ plasma)</td>
<td>$\omega^2 = \omega_{pe}^2 + 3v_T^2k^2$</td>
<td>e/s</td>
<td>$\vec{E} \parallel \vec{k}$</td>
</tr>
<tr>
<td>IAW (ion acoustic)</td>
<td>$\omega^2 = c_a^2k^2$</td>
<td>e/s</td>
<td>$\vec{E} \parallel \vec{k}$</td>
</tr>
</tbody>
</table>

Matching: $\omega_1 = \omega_2 + \omega_3 \quad k_1 = k_2 + k_3$

$$\vec{E}_j = a_j(x,t) \sqrt{\frac{2\omega_j}{\epsilon_0}} \epsilon_j e^{(k_jx - \omega_jt)}$$

$$|a_j|^2 = \frac{\epsilon_0}{2\omega_j} |E_j|^2 = \text{action density}$$

$$\frac{Da_1}{D_1t} = -Ka_2a_3$$
$$\frac{Da_2}{D_2t} = K^*a_1a_3^*$$
$$\frac{Da_3}{D_3t} = K^*a_1a_2^*$$

$$\frac{D}{D_jt} = \frac{\partial}{\partial t} + v_gj \frac{\partial}{\partial x} + v_j$$

- $\omega_{las} \approx \omega_{SBS} = 5.59$
- $\omega_{SRS} = 4.49$
- $\omega_{EPW} = 1.1$
- $\omega_{LDI} = 0.016$
- $\omega_{SBS-IAW} = 0.00915$
- $k_{las} \approx -k_{SBS} = 5.5$
- $k_{SRS} = -4.38$
- $k_{EPW} = 10.1$
- $k_{LDI} = 19.4$
- $k_{SBS-IAW} = 11$

Laser-Modified Equilibrium: Electromagnetic Fields

\[ v_{ye} \text{ at } \omega_p t = 100 \]

\[ E^- \text{ at } \omega_p t = 100 \]

\[ |v_{ye}(k)|^2 \text{ at } \omega_p t = 100, x\omega_p/c = 20 - 80 \]

\[ |E^-(k)|^2 \text{ at } \omega_p t = 100, x\omega_p/c = 20 - 80 \]
Laser-Modified Equilibrium: Electrostatic Fields

\[ n_e - n_{e0} \text{ at } \omega_p t = 100 \]

\[ |n_e(k)|^2 \text{ at } \omega_p t = 100, \ x\omega_p/c = 20 - 80 \]

\[ E_x \text{ at } \omega_p t = 100 \]

\[ |E_x(k)|^2 \text{ at } \omega_p t = 100, \ x\omega_p/c = 20 - 80 \]
Reflected Light at $x = 0$

**Instantaneous Reflectivity (Avg.=3.1%)**

<table>
<thead>
<tr>
<th>$E^-(\omega)^2$</th>
<th>at $x = 0$, $t\omega_p = 200 - 400$</th>
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<td>$</td>
<td>E^-(\omega)^2</td>
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Frequency of reflected light $E^-(x = 0)$ vs. time

- **Upshift in SRS frequency** at later times. Related to downshift in EPW frequency due to trapping?
- **SBS develops** at later times. Not present in run with stationary ions (see below).

Reflected Light at \( x = 0 \), Stationary Ions

Instantaneous Reflectivity (Avg.: 3.4%)

\[
\frac{\langle |E|^2 \rangle_t}{\langle |E|^2 \rangle_{t \to x}} \text{ vs } \omega p t
\]

\[ |E^{-}(\omega)^2| \text{ at } x = 0, \ t\omega_p = 200 - 400 \]

\[ |E^{-}(\omega)^2| \text{ at } x = 0, \ t\omega_p = 800 - 1000 \]

Longitudinal Electric Field $E_x$ vs. $t$

$E_x$ at $x\omega_p/c = 40$

RMS $E_x(t)$ at $x\omega_p/c = 40$

$|E_x(\omega)|^2$ at $x\omega_p/c = 40, \omega_p t = 200 - 400$

$|E_x(\omega)|^2$ at $x\omega_p/c = 40, \omega_p t = 800 - 1000$

Longitudinal Electric Field $E_x$ vs. $x$

**RMS $E_x$ at $\omega_p t = 400$**

<table>
<thead>
<tr>
<th>$E_x(k)$</th>
<th>$\omega_p t = 400$, $x\omega_p/c = 30 - 120$</th>
</tr>
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**$E_x$ at $\omega_p t = 900$**

<table>
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<tr>
<th>$E_x(k)$</th>
<th>$\omega_p t = 900$, $x\omega_p/c = 30 - 120$</th>
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</table>

Evolution of Trapped Distribution

Contour Plots of $f_e$ (log scale)

Trapped $f_e$ during first SRS burst

Contour Plot of $f_e$ at $\omega_p t = 400$

Spatially-averaged $\langle f_e \rangle_x$

Maxwellian fit: “beam”

$\langle f_e \rangle_x$ — Maxwellian fit: “beam”

Wave-Breaking: Possible Limit to SRS?

- **Wave-breaking threshold**: Travelling-wave solutions to electrostatic warm-fluid equations \( n(kx - \omega t) \), \( u(kx - \omega t) \) do not exist with electric field amplitudes \( E > E_{\text{max}} \).


\[
E_{\text{max}}^2 = 1 - \frac{1}{3}\beta - \frac{8}{3}\beta^{1/4} + 2\beta^{1/2}
\]

\[
\beta \equiv \frac{T_e}{T_0} \quad T_0 \equiv \frac{1}{3}m_e e V \left(\frac{\omega}{ck}\right)^2 = 2 \text{ keV}
\]

**\( E_{\text{max}} \) vs. \( T_e \) of SRS EPW for run parameters**

![Graph showing the relationship between \( E_{\text{max}} \) and \( T_e \) for SRS EPW with run parameters. The graph shows a curve that decreases as \( T_e \) increases, with \( E_{\text{max}} \) ranging from 0.12 to 0.02 as \( T_e \) goes from 0 to 2000 eV. The graph includes annotations for \( T_e = 350 \text{ eV} \) and \( E_{\text{max}} = 0.0255 \).]
Scaling of Bounce Frequency with Amplitude of $E_x$

$$\omega_B \equiv \sqrt{\frac{eE_x k_{EPW}}{m_e}}$$

**Bounce Freq. vs. $E_x$**

![Graph showing the relationship between bounce frequency and $E_x$. The graph plots $\omega_B/\omega_p$ on the y-axis against $E_x$ on the x-axis. Key values highlighted include $\omega_B = \gamma_{SRS}$ at $E_x = 0.01$, and $E_x = E_{x,\text{wavebreak}}$ at $\omega_B = 0.51$.](image-url)
Ion Density Fluctuations vs. $x$

$n_i - n_{i0}$ at $\omega_p t = 450$

$|n_i(\omega)|^2$ at $\omega_p t = 450$, $x\omega_p/c = 30 - 120$

$n_i - n_{i0}$ at $\omega_p t = 1000$

$|n_i(\omega)|^2$ at $\omega_p t = 1000$, $x\omega_p/c = 30 - 120$
Run too short for ion acoustic wave to go through several cycles, e.g.:

\[ c_a = 8.3 \cdot 10^{-4} \quad k_{IAW-SBS} = 11 \quad \rightarrow \quad \tau = \frac{2\pi}{c_a k_{SBS}} = 688 \]
Future Work: Parallelize, $1\frac{1}{2}$ -D Code

Preliminary results from $1\frac{1}{2}$ -D code that treats $v_y \ll c$ kinetically: $f_s(x, p_x, v_y, t)$.

$$\frac{\partial f_s}{\partial t} + v_x \frac{\partial f_s}{\partial x} + q_s (E_x + v_y B_z) \frac{\partial f_s}{\partial p_x} + \frac{q_s}{\gamma m_s} (E_y - v_x B_z) \frac{\partial f_s}{\partial v_y} = 0$$

**$n_e$ at $\omega_p t = 8$**

**$E_y$ at $\omega_p t = 8$**