Cone-Guided Fast Ignition with Imposed Magnetic Fields


*LLNL*

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Our fast ignition modeling approach

**Explicit PIC** modeling of short-pulse laser-plasma interaction: A. J. Kemp, L. Divol

**Rad-hydro:** fuel assembly in hohlraum, around cone: H. D. Shay, M. Tabak, D. Ho

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**Transport** modeling of fast electron heating: hybrid PIC code Zuma coupled to Hydra

Subject of this talk
Some trick, like imposed magnetic fields, is needed to achieve fast ignition with a realistic, divergent fast electron source

- Fast electron source generated by short-pulse laser – characterized by PIC sims:
  - Energy spectrum has two temperature components, many electrons too energetic to stop in DT hotspot
  - Angle spectrum is divergent – serious challenge!

- Transport modeling: hybrid PIC code Zuma coupled to Hydra rad-hydro code

- Imposed uniform axial magnetic fields > 30 MG mitigate divergence
  - Can be produced in an implosion with seed field ~ 50 kG

- Magnetic mirroring in non-uniform field prevents fast electrons from reaching fuel

- Hollow magnetic pipe can prevent mirroring
Fast electron source distribution found from explicit PIC laser-plasma simulations with PSC code (A. Kemp, L. Divol)

Electron density (ripples due to laser absorption)

- 3D Cartesian run, 1 µm laser wavelength, pre-plasma with $n_e \sim \exp[ z / 3.5 \ \mu m ]$
- Intensity at vacuum focus ($z = 10 \ \mu m$): $I_{\text{las}}(r) = I_0 \exp[-(r/18.3 \ \mu m)^8]$
- $I_0 = 1.37 \ \text{E}20 \ \text{W/cm}^2$

Extraction box: all fwd-going electrons with kin. en. > 0.5 MeV

Source box: fast electrons excited here in equivalent transport simulation

$$f(E,\theta) = f_E(E) \ast f_\theta(\theta) \quad \text{factorized}$$

$$I_{\text{fast}}(r,t) = 0.52 \ast I_{\text{las}}(r,t)$$
PIC fast electron energy spectrum is quasi two-temperature

\[ \frac{dN}{d\varepsilon} = \frac{1}{\varepsilon} \exp[-\varepsilon / 0.19] + 0.82 \exp[-\varepsilon / 1.3] \quad \varepsilon = \frac{E}{T_p} \]

We scale \( dN/d\varepsilon \) with ponderomotive temperature [S. C. Wilks et al., Phys. Rev. Lett. (1992)]

\[ \frac{T_{\text{pond}}}{m_e c^2} = \left[ 1 + a_0^2 \right]^{1/2} - 1 \sim a_0 \equiv \sqrt{\frac{I_{\text{las}} \lambda^2}{1.37 \cdot 10^{18} \text{ W cm}^{-2} \mu\text{m}^2}} \]

\[ \lambda = 1 \mu\text{m}, \quad I_0 = 1.37 \text{ E20 W/cm}^2 \rightarrow T_{\text{pond}} = 4.63 \text{ MeV} \]

- Same spectrum in source and extraction box

- Ignition DT hot spot: \( \rho \Delta z \sim 1.2 \text{ g/cm}^2 \). Removes \( \sim 1.4 \text{ MeV} \) from a fast electron (neglecting angular scatter)
  - Spectrum is too energetic to stop in hot spot
PIC fast electron angle spectrum is very divergent

solid angle spectrum in source box: 
\[ \frac{dN}{d\Omega} = \exp \left[ -\left( \frac{\theta}{\Delta \theta} \right)^4 \right] \]

\( \Delta \theta = 90^\circ \) needed to agree with PIC results

### Solid angle spectrum

- Transport source box, \( \Delta \theta = 90^\circ \)
- PIC extraction box
- Transport extraction box

### Average polar angle

Zuma-Hydra runs used for

<table>
<thead>
<tr>
<th>( \Delta \theta )</th>
<th>( &lt;\theta&gt; )</th>
<th>artifically collimated source</th>
<th>realistic PIC source</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>6.9°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90°</td>
<td>52°</td>
<td></td>
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Zuma: Hybrid PIC code (D. J. Larson)

• Field and background dynamics simplified to eliminate light and plasma waves:
  valid for $\omega << \omega_{\text{plasma}}, \omega_{\text{laser}}$, $k << k_{\text{laser}}, 1/\lambda_{\text{Debye}}$

• Relativistic fast electron particle push: $\vec{F} = -e(\vec{E} + \vec{v} \times \vec{B})$

• Fast e- energy loss (drag) and angular scattering: formulas of Solodov, Davies

• $\vec{J}_{\text{return}} = -\vec{J}_{\text{fast}} + \mu_0^{-1} \nabla \times \vec{B}$
  Ampere's law without displacement current

• Electric field given by massless momentum equation for background electrons:

$$m_e \frac{d\vec{v}_{eb}}{dt} = -e\vec{E} + \ldots = 0$$

$$\rightarrow \vec{E} = \eta \cdot \vec{J}_{\text{return}} - e^{-1} \vec{\beta} \cdot \nabla T_e - \frac{\nabla p_e}{en_{eb}} - \vec{v}_{eb} \times \vec{B}$$

$$\eta = \text{resistivity from Lee-More-Desjarlais}$$

• $\vec{E} = \eta \vec{J}_{\text{return}}$
  Simple Ohm's law, used for this talk's runs

• $\vec{J}_{\text{return}} \cdot \vec{E}$
  Ohmic heating

• $\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$
  Faraday's law
Hybrid PIC transport code Zuma coupled to rad-hydro code Hydra (M. M. Marinak, D. J. Larson, L. Divol)

- Both codes run in cylindrical R-Z geometry on fixed Eulerian meshes (which can differ)

- Data transfer done via files generated by Overlink code [J. Grandy et al.]

- Typical run: 20 ps transport (Zuma + Hydra), then 180 ps burn (just Hydra)
  - 2-3 wall-time hours on 40 cpu’s

- Hydra details: IMC photonics, neutron deposition turned off, no MHD package used
ignition-scale toy target with carbon cone

- Ideal burn-up fraction $f = \frac{\rho R}{(\rho R + 6)} = \frac{1}{3}$
- Ideal fusion yield = $338 \text{ MJ} \times \text{Mass [mg]} \times f = 64.4 \text{ MJ}$
- Optimal e-beam ignition energy [Atzeni et al., PoP 2007] $140 \text{ kJ} / (\rho/100 \text{ g/cc})^{1.85} = 8.7 \text{ kJ}$
- Beam intensity = $I_0 \exp(-0.5*(r/23)^8)$ HWHM: $r = 24 \mu\text{m}$
- $527 \text{ nm (2ω)}$ wavelength laser: lowers $T_{\text{pond}} \sim \lambda$

DT density

- Cone: 30 g/cc carbon
- DT fuel

- $\rho > 100 \text{ g/cc}$: $\rho R = 3.0 \text{ g/cm}^2$
- mass = 0.572 mg

DT density vs. radius

- $t \text{ [ps]} = 2.07497$
- $\rho \text{ [g/cm}^3\text{]}$

Beam intensity vs. time

- $e^\frac{-1}{2}(r/23)^8$
PIC-based source divergence gives prohibitive ignition energies; dramatically reduced if source is artificially collimated

87 kJ ignition energy
10x ideal value:
divergence, spectrum too energetic, pulse and spot un-optimized

\[
\Delta \theta = 10^\circ \quad \text{(artificial collimation)}
\]

\[
\Delta \theta = 90^\circ \quad \text{(PIC-based)}
\]

max = 1307 Gbar
max = 235 Gbar
Adding an initial, uniform, axial magnetic field $B_z$ reduces ignition energy to that of artificially collimated beam

$$E_{\text{fast}} = \text{fast e}^- \text{ energy \ [kJ]}$$

$$r_{\text{Le}} \propto \frac{\gamma \beta}{B} = \frac{33.4 \ \mu m}{B_{\text{MG}}} \left[ W^2_{MV} + 1.02 W_{MV} \right]^{1/2}$$

For a 2 MeV e- (roughly optimal to deposit energy in hot spot), $r_{\text{Le}} = 82$ um / B [MG]

$$r_{\text{Le}} = \text{spot radius (24 \ um)} \text{ for } B = 3.4 \ \text{MG}: \text{lower bound on when } B \text{ fields matter}$$

Rad-hydro-MHD studies of B field compression have been started by H. D. Shay, M. Tabak

Omega experiments show compression of 50 kG seed B field in cylindrical implosions\textsuperscript{1} to 30-40 MG, and in spherical implosions\textsuperscript{2} to 20 MG

\textsuperscript{1}J. P. Knauer, Phys. Plasmas 17, 056318 (2010) \quad \textsuperscript{2}P. Y. Chang et al., talk J05-2, APS-DPP 2010
Implosion can compress magnetic field in DT, but short-pulse LPI will likely happen in the seed field

- Cone outer surface compressed
- Cone inner surface doesn’t move – shock break-out would fill cone
- B field in vacuum region is uncompressed; may increase over seed field due to diffusion

Nested conductors:
Field compressed between conductors, but not inside inner one

Flux $B_z \times r^2$ conserved inside conductor, unless field diffuses due to resistivity
Axial variation in magnetic field strength reduces hot-spot heating due to magnetic mirroring

\[ B_z(r, z) = B_{z0} + (B_{z1} - B_{z0})G(r)H(z) \]

\[ H(z) = \left(1 + \left(\frac{z - z_0}{\Delta z}\right)^2\right)^{-1} \]

\[ G(r) = \exp\left[-(r / 50 \ \mu m)^8\right] \]

Axial variation in \( B_z \) gives rise to \( B_r \), to satisfy \( \text{div} \ B = 0 \). Leads to mirror force in \( z \) direction.
Magnetic mirroring in cylindrical B field

\[ \frac{d\vec{p}}{dt} = q\vec{v} \times \vec{B} \]

\[ \vec{B} = B_z(z)\hat{z} + B_r(r,z)\hat{r} \]

\[ \omega_{cz} = \frac{qB_z}{\gamma m} \]

\[ \nabla \cdot \vec{B} = 0 \quad \Rightarrow \quad B_r = -\frac{r}{2} \frac{\partial B_z}{\partial z} \]

\[ \frac{d^2r}{dt^2} = -\frac{d}{dr} U_r \quad U_r = \frac{l_0^2}{2r^2} + \frac{r^2}{8} \omega_{cz}^2 \]

equilibrium: \[ dU_r / dr = 0 \quad \Rightarrow \quad r_E = \left| \frac{2l_0}{\omega_{cz}} \right|^{1/2} \]

Axial motion:
\[ \frac{d^2z}{dt^2} = \frac{1}{8} \left( \sigma r_E^2 - r^2 \right) \frac{\partial}{\partial z} \omega_{cz}^2 \]
\[ \sigma = \text{sign}(l_0 \omega_{cz}) \]

Mirroring in increasing \( B_z \):
- Fight between \( dB_z/dz \) and decreasing radius
- Adiabatic invariant for slowly changing \( B_z \):
  "loss cone" \( \tan^2 \theta_L = B_{z0} / (B_{z1} - B_{z0}) \)
- Loss cone not accurate for rapidly-varying \( B_z \):
  Large \( B_r \) can mirror particles with \( v_{\text{perp}}(t=0) = 0 \)
  (always in adiabatic loss cone)

Radial motion:
Axial motion:
We can circumvent mirroring with “magnetic pipe:”

$B_{z_0}$ peaks off-axis

- Run with $B_{z_0} = 90$ MG, $E_{\text{beam}} = 87$ kJ ignites

- Using $B_{z_0} = 60$ MG, or narrower in z, or $E_{\text{fast}} = 43.4$ kJ all fail (<270 kJ fusion yield).

- Artifically collimated beam ($\Delta \theta = 10^\circ$) requires $E_{\text{fast}} = 87$ kJ to ignite.

Very little backward-going $e-$, unlike mirroring cases
Imposed magnetic fields may circumvent large fast-electron divergence for fast ignition, but mirroring is an issue

- Artificially collimated e- beam: ignition $E_{\text{fast}} = 87$ kJ
- Realistic PIC beam divergence: ignition $E_{\text{fast}} \sim$ MJ’s
- Uniform initial axial magnetic field $> 30$ MG: ignition $E_{\text{fast}} = 100$ kJ
- Non-uniform field peaking in fuel: fast e- reflected by mirror force
- Magnetic pipe: hollow radial profile: can recover ignition $E_{\text{fast}} = 87$ kJ

How can we assemble such fields in an implosion?
• Backup slides beyond here
We are pursuing fast ignition for high gain and inertial fusion energy.

Long-pulse compression laser \( \sim 1 \text{ MJ} \)

\( \rho \sim 300-500 \text{ g/cm}^3 \)

\( \rho_r > 2 \text{ g/cm}^2 \)

Final state of compressed target

| 200 \( \mu \text{m} \) |

Cold dense fuel

Low density corona

Gold (or other mid/high Z) cone

"Transport region:" \( \sim 50-100 \mu \text{m}; \) Subject of this talk.

Short-pulse laser produces fast electrons

10-20 ps pulse

\( < 100 \text{ kJ} \) (to ever be built)

Power \( \sim 5-10 \text{ PW} \)

\( \sim 50 \mu \text{m} \) focal spot (FWHM)

Isochoric ignition hot-spot: $T_{\text{ion}} > 4$ keV and $\rho * R * T_{\text{ion}} > 5$ g cm$^{-2}$ keV


$X_h$ = hot-spot value; $\rho_c$ = density of surrounding cold fuel. $\rho_c = \rho_h$ for isochoric.

$\rho_h R_h T_h (\rho_c / \rho_h)^{1/2}$ [g cm$^{-2}$ keV]

$\rho * R * T_{\text{ion}} = \text{max. at end of e- source pulse, centered on peak ion pressure.}$
We can circumvent mirroring with “magnetic pipe:”

**B**\(_{z0}\) peaks off-axis

\[ B_z(r, z) = B_{z0} + (B_{z1} - B_{z0})G(r)H(z) \]

\[ G(r) = \exp\left[-\left(\frac{r-r_0}{\Delta r}\right)^4\right] \]

\[ H(z) = \left[1 + \left(\frac{z-z_0}{\Delta z}\right)^2\right]^{-2} \]

Field type 2a: \(B_{z0} = 0.1\) MG, \(B_{z1}\) varies, \(z_0 = 20, \Delta z = 50, r_0 = 30, \Delta r = 10\)

Field type 2b: same as 2a, but \(\Delta z = 100\)
Magnetic field evolution governed by MHD frozen-in law

\[ \partial_t \vec{B} = -\nabla \times \vec{E} \]
\[ \vec{E} = -\vec{v}_e \times \vec{B} + \eta \vec{J}_e \]
\[ \nabla \times \vec{B} = \mu_0 \vec{J}_e \]

Cylindrical geometry:
\[ \vec{B} = B_z(r,t) \hat{z} \]
\[ \eta = \eta(r,t) \]
\[ \vec{v}_e = v_r(r,t) \hat{r} \]

Let \( \frac{dr_i}{dt} = v_r(r_i,t) \) follows plasma electron flow

Then \[ \frac{d\psi}{dt} = \frac{2\pi}{\mu_0} \left( r \eta \frac{\partial B_z}{\partial r} \right) \bigg|_{r_i}^{r_2} \]

Frozen-in law: magnetic flux between two surfaces moving with the plasma electrons changes only due to magnetic diffusion.
Mirroring with non-uniform imposed B-fields: effective beam energy partly follows mirror scaling

<table>
<thead>
<tr>
<th>B field type</th>
<th>$B_{z,fuel} / B_{z,exc}$</th>
<th>mirror $\Phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>0.66</td>
<td>0.52 (mid)</td>
</tr>
<tr>
<td>F</td>
<td>0.34</td>
<td>0.24 (worst)</td>
</tr>
<tr>
<td>G</td>
<td>0.88</td>
<td>0.76 (best)</td>
</tr>
</tbody>
</table>

black: uniform $B_{z0}=50$ MG; mirror $\Phi = 1$. 
Evidence of mirroring with non-uniform imposed B-fields: reflected fast electrons

\[ B_{z0} = 0, \ E_{\text{beam}} = 173 \ \text{kJ} \]

more mirroring

\[ B_{z0} = 50 \ \text{MG}, \ E_{\text{beam}} = 87 \ \text{kJ} \]

field type G, \[ E_{\text{beam}} = 173 \ \text{kJ} \]

field type E, \[ E_{\text{beam}} = 173 \ \text{kJ} \]

field type F, \[ E_{\text{beam}} = 173 \ \text{kJ} \]

\[ |J_{\text{beam}}| \ [\text{Amp/m}^2], \ t = 10 \ \text{ps} \]
Magnetic mirroring generalities (fully relativistic)

- \( \text{div } B = 0 \) implies \( B_r(r,z) = -(r/2) \frac{dB_z}{dz} \)
- Mirroring due to \( z \) force on a particle: \( F_z = q \mathbf{v}_\perp \times B_r \)
- Adiabatic limit: \( \left| \frac{1}{B} \frac{dB}{dt} \right| \ll \text{cyclotron freq.} \)
- Magnetic moment = adiabatic invariant: \textit{not} exactly conserved, but change is small
  \[
  \mu = \oint p_\perp \cdot d\mathbf{l} = \pi \frac{c}{e} \frac{p_\perp^2}{B_z} \quad \rightarrow \quad \frac{v_\perp^2}{B_z} = \text{const.}
  \]
  \[
  v_z^2 + v_\perp^2 = \text{const.} \quad \rightarrow \quad v_z^2 = v_{z0}^2 + v_{\perp0}^2 \left( 1 - \frac{B_z}{B_{z0}} \right)
  \]
- Loss cone bad in MFE mirror machine, but good for us: these e- reach the fuel
  \[
  \text{loss cone: } \tan \theta_{0L} = \frac{v_\perp}{v_z} = \left[ \frac{B_{z0}}{B_{z1} - B_{z0}} \right]^{1/2}
  \]
Mirroring with our electron source

Loss cone angle vs. mirror ratio

\[ \tan \theta_{0L} = \left( \frac{B_{z0}}{B_{z1} - B_{z0}} \right)^{1/2} \]

mirror ratio = \( \frac{B_{z0}}{B_{z1}} \)

- e- source: \( \frac{d^2 N}{dE d\theta} = \frac{dN}{dE} \cdot \frac{dN}{d\theta} \)

\[ \frac{dN}{d\theta} = \sin \theta \exp \left[ -\left( \frac{\theta}{90 \text{ deg.}} \right)^4 \right] \]

Number in loss cone: \( F(\theta) = \int_{0}^{\theta} d\theta \frac{dN}{d\theta} \)

loss-cone fraction: \( \Phi = \frac{F(\theta)}{F(\pi / 2)} \)

Loss cone fraction of our \( dN/d\theta \)

vs. mirror ratio

\[ \Phi = 0.65 \frac{B_{z0}}{B_{z1}} \]