DEPLETE - a code for rapid assessment of backscatter activity

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## Hierarchy of Laser-Plasma Interaction (LPI) codes

<table>
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<tr>
<th>Code</th>
<th>Description</th>
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<tr>
<td>NEWLIP</td>
<td>Linear kinetic gain calculation along 1-D ray profile; steady-state.</td>
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<tr>
<td>DEPLETE</td>
<td>Like NEWLIP, but solves for pump and scattered intensities with 1-D model of thermal fluctuations.</td>
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<tr>
<td>SLIP</td>
<td>3-D, steady-state, kinetic coefficients.</td>
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<tr>
<td>pF3D</td>
<td>Laser propagation; enveloped laser and daughter waves; time evolution; 1-, 2-, or 3-D (patch, letterbox, whole beam, ...).</td>
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<tr>
<td>Kinetics</td>
<td>Full plasma physics (but small volumes).</td>
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**DEPLETE** calculates the laser and backscattered intensities, in steady state, along a 1-D ray profile. It solves for a set of scattered-wave frequencies, with physical noise and pump depletion. Kinetic formulas are used for coupling and Thomson.
NEWLIP: finds linear backscatter gains for rad-hydro simulations; quick-and-dirty estimate

- NEWLIP - Yorick code by E. A. Williams; finds gain \( G(\omega_1) \) along many (~hundreds) of rays, to *quickly* assess target’s backscatter risk.

\[
\partial_s i_1(s, \omega_1) = -\alpha i_1 \quad \alpha \equiv \frac{1}{4} \frac{f}{\eta_0} \frac{v_{os,0}^2}{c^2} \frac{k_2^2}{|k_{1p}|} \sum \chi_e \left( 1 + \chi_I \right) / \epsilon
\]

\( s = \) distance along ray path;

\[
I_1 = \int d\omega_1 i_1
\]

Solved for many \( \omega_1 \)'s.

\[
\chi_I = \sum_{i=\text{ions}} \chi_i \quad \chi = \chi_e + \chi_I = \text{susceptibility} \quad \epsilon = 1 + \chi = \text{dielectric}
\]

- Plasma waves in the strongly-damped limit:

\[
\frac{n_2}{n_e} = \frac{1}{2} \left| \frac{\chi_e}{\epsilon} \right| \left( k_2 \lambda_{De}^2 \right) \frac{v_{os,0} v_{os,1}}{v_{Te}^2}
\]

\[
n_{2e} = (1 + \chi_I) n_2
\]

- Linear gain:

\[
i_1(s_L) = i_1(s_0) e^G; \quad G \equiv \int_{s_0}^{s_L} ds \ \alpha(s)
\]

Linear gain \( G \) is the main “product” of NEWLIP.
DEPLETE equations: solve for pump and scattered intensities

\[
\begin{align*}
\partial_s I_0(s) &= -\kappa_0 I_0 - \int d\omega_1 \frac{\omega_0}{\omega_1} I_0 \cdot (\tau_1 + \Gamma_1 i_1) \\
\partial_s i_1(s, \omega_1) &= \kappa_1 i_1 - \Sigma_1 - I_0 \cdot (\tau_1 + \Gamma_1 i_1)
\end{align*}
\]

- **inverse bremsstrahlung:**
  \[\kappa_i \equiv \frac{\omega_{pe}^2 \nu_{ei}}{\omega_i^2 \nu_{gi}}\]
- **brem. noise:**
  \[\Sigma_1 = \int f^{-1} \Omega_c \kappa_1 \frac{v^2_i}{c^2} B_v\]
- **Thomson coupling:**
  \[\tau_1 \equiv \frac{K_T}{|\epsilon|^2}, \quad K_T = \frac{\Omega_c}{\sqrt{2\pi}} n_e r_e^2 \omega_0 \frac{g_T}{\omega_{pe} k_2 \lambda_{De}}\]
- **Coupling:**
  \[\Gamma_1 = \frac{K_G}{|\epsilon|^2}, \quad K_G \equiv f \frac{2\pi r_e}{m_e c^2} \frac{1}{\omega_0} \frac{k_2^2}{k_{0p} |k_{1p}|} g_G\]

**scattered light**

\[\Omega_c \equiv 2\pi (1 - \cos \theta_c) \approx \frac{\pi}{4F^2}\]

**F-cone:**
\[\cos \theta_c \equiv \left[1 + \frac{1}{4F^2}\right]^{-1/2} \approx 1 - \frac{1}{8F^2}\]

**whole-beam focusing:**
\[f \equiv \frac{\text{area}(s)}{\text{area}(s_{focus})}\]

Intensities are: \[\text{[total power in ray]} / \text{[focal spot area]}\].

- **Bremsstrahlung:**
  \[\nu_{gi} = \text{blackbody}\]
- **Thomson:**
  \[B_v = \text{blackbody}\]
- **Coupling:**
  \[1 + \chi I |\chi_e|^2 e^{-\zeta_e^2} + |\chi_e|^2 \sum_i \frac{v_{Ti}}{v_{Te}} e^{-\zeta_i^2}\]
DEPLETE numerics: shoot on $I_0$ (s=wall), split step

- **Shooting:** Two-point boundary-value problem. March from wall to LEH, varying $I_0$ (wall) until $I_0 (LEH)$ is close enough to known value.

  - **Split step:**

  $\begin{bmatrix} I_0^{n-1} \\ i_1^{n-1} \end{bmatrix}_{s_{n-1}} \rightarrow \begin{bmatrix} I_0^n \\ i_1^n \end{bmatrix}_{s_n}$

  \[
  [I_0, i_1]_{s_{n-1}} = B_{1/2} \cdot C_1 \cdot B_{1/2} \cdot [I_0, i_1]_{s_n}
  \]

  $B_{1/2} =$ bremsstrahlung for a half-step:

  \[
  \partial_s I_0 = -\kappa_0 I_0 \\
  \partial_s i_1 = \kappa_1 i_1 - \Sigma_1
  \]

  $C_1 =$ coupling-Thomson for whole step:

  \[
  \partial_s I_0 = -\int d\omega_1 \frac{\omega_0}{\omega_1} I_0 \cdot (\tau_1 + \Gamma_1 i_1) \\
  \partial_s i_1 = -I_0 \cdot (\tau_1 + \Gamma_1 i_1)
  \]

  Run time dominated by evaluating Z functions, not ODE solving (even with shooting).
Coupling-Thomson step: Analytical solution for narrow resonances

Coupling-Thomson step over a single s cell: hold $I_0$ constant, solve for $i_1$ for every $\omega_1$, then update $I_0$ conservatively (Manley-Rowe).

\[
\partial_s i_1 = -I_0 \cdot (\tau_1 + \Gamma_1 i_1)
\]

\[
\Gamma_1 = \frac{K_T}{|\epsilon|^2} \quad \tau_1 \equiv \frac{K_{\tau}}{|\epsilon|^2}
\]

- **Problem:** narrow resonances in coupling and Thomson coefficients hard for standard ODE solvers (e.g. Runge-Kutta).

- **Solution:** The resonance occurs when Re[$\epsilon$] = 0 in the denominator; $\epsilon$ itself varies slowly, so linearize $\epsilon$ in a cell and analytically solve.

\[
\epsilon \approx \epsilon^{n-1/2} + \partial_s \epsilon^{n-1/2} \cdot (s - s_{n-1/2})
\]

\[
\partial_z i_1 = -\frac{B_\tau + B_\Gamma i_1}{1 + z^2}
\]

\[
i_1^{n-1} = (i_1^n + \beta) e^{B_\Gamma \Delta \omega} - \beta
\]

\[
\beta \equiv \frac{B_\tau}{B_\Gamma}
\]

\[
\Delta w \equiv \tan(\omega_n) - \tan(\omega_{n-1})
\]

E. A. Williams uses a similar technique for finding gains in NEWLIP ("ratint").
Properties of DEPLETE

- Fast! Almost as fast as NEWLIP. Most time spent evaluating kinetic Z functions.
- Works along 1-D ray profiles (rad-hydro usually treats lasers via rays).
- 1-D noise: 3-D bremsstrahlung noise taken over beam F-cone.
- Gives scattered-wave intensities:
  - Measurable, unlike gain.
  - Allows for assessment of nonlinearities (e.g. trapping, LDI, inflation).
  - Re-absorption of scattered waves done.
- Pump depletion included.
- Kinetic description - same as NEWLIP, better than pF3D (some fluid approximations).
- Different scattered frequencies $\omega_1$ handled simultaneously:
  - No enveloping around a carrier wave, as in pF3D.
  - Different $\omega_1$’s treated incoherently (no spectral leakage; physical?).

- DEPLETE lacks some physics in pF3D:
  - Steady-state - no time evolution.
  - 1-D: no transverse gradients, beam intensification.
  - no speckle physics, no beam smoothing (but we have ideas).
  - no plasma-wave advection (strong damping limit).
  - DEPLETE model not strictly valid for absolute instability.
Tests of DEPLETE on “clean” profile: linear gradients, just SRS

pF3D run: 3-D plane wave; no transverse pump structure like speckles.

**Graphs:**
- $n_e/n_c$ vs. $s$ (cm)
- $Te$ (keV) vs. $s$ (cm)
- $SRS$ reflectivity vs. $l_0$ ($10^{15}$ W/cm$^2$)

$L_0 = 8.7 \times 10^{14}$
“Clean” profile test: scattered-light spectrum from pF3D and deplete have similar shape

In the linear regime, the spectra are quite similar.

\[ I_0 = 3 \times 10^{14} \]

In the saturated regime, pF3D has more pump depletion, less scattering from high density.

\[ I_0 = 1.17 \times 10^{15} \]
“Clean” profile test: NEWLIP gain vs. DEPLETE boost

- Increases confidence in gain, and is a “sanity check” on DEPLETE.
- But NEWLIP doesn’t give intensities.

\[
\text{DEPLETE "Boost"} = \log\left(\frac{\text{scattered spectrum with brem., Thomson, and coupling}}{\text{scattered spectrum with just bremsstrahlung (i.e. no laser)}}\right) = \frac{\text{"backscatter"}}{\text{"noise"}}
\]

\[
I_0 = 6 \times 10^{14} \text{ W/cm}^2 \\
I_0 = 1 \times 10^{15} \text{ W/cm}^2 \\
I_0 = 1.34 \times 10^{15} \text{ W/cm}^2
\]

- Boost > gain due to Thomson
- Boost < gain due to pump depletion (and some absorption)

D. J. Strozzi: Anomalous Absorption 2007: p. 10
Sample profile: inner beam (23 deg.) Be ray, 270 eV point design at time of peak laser power

$\frac{n_e}{n_c}$

LEH wall

Transmission

solid: pump with
inv. brem. +
pump dep.

dashed: pump with just
inv. bremsstrahlung

SRS
SBS

escaping
SRS, SBS

i$_1$ SRS (log scale)
i$_1$ SBS (log scale)

$\frac{i}{i_0}$

$\lambda_1$, $\lambda_0$ (A ng)
DEPLET and NEWLIP gain have similar reflected-light spectra

$$\text{DEPLET} \quad \text{“Boost”} = \log \left[ \frac{\text{scattered spectrum with brem., Thomson, and coupling}}{\text{scattered spectrum with just bremsstrahlung (i.e. no laser)}} \right] = \frac{\text{“backscatter”}}{\text{“noise”}}$$
DEPLETE and pF3D patch agree pretty well (on a log scale); speckle effects may enhance SRS

pF3D patch: “ray with speckles:” 3D, whole ray path, a few speckles in transverse directions.

pF3D has more SRS: probably due to higher gain in high-intensity speckles.
pF3D has less SBS: probably due to more pump depletion.

The decent agreement of DEPLETE and pF3D scattered intensities validates DEPLETE’s 1-D model of 3-D noise.
The plasma-wave amplitude predicted by DEPLETE can be compared with nonlinearity thresholds

\[ \frac{n_2}{n_e} = \frac{1}{2} \left| \frac{\chi e}{\epsilon} \right| \left( k^2 \lambda_D e \right)^2 \frac{v_{os,0} v_{os,1}}{v_{Te}^2} \]

\[ \frac{n_2}{n_{e0}} = \left[ \int d\omega_1 \delta_2 \right]^{1/2} \]

\[ \tau_B \omega_{pe} = 2\pi \left[ \frac{n_{e0}}{n_2} \right]^{1/2} \]

SRS electron plasma wave: too weak for trapping nonlinearities
Conclusions and Future Prospects

Conclusions:
• DEPLETE provides a 1-D, steady-state, ray-based, linear kinetic calculation of backscatter:
  – Pump depletion, re-absorption, 1-D physical noise included.
• Compares well with NEWLIP gains.
• pF3D comparisons are promising; need to include speckle effects in DEPLETE for better agreement.

Future prospects:
• DEPLETE can be incorporated into rad-hydro codes:
  – ray-based (just like rad-hydro) and computationally fast (~secs. per ray).
  – An effective absorption coefficient, calculated from the DEPLETE solution (including scattered and plasma wave intensities) can replace the bremsstrahlung damping rate in the rad-hydro code.
• DEPLETE gives plasma wave intensities, which can be compared to nonlinearity thresholds (Langmuir decay instability, trapping, kinetic inflation, Langmuir wave self-focusing).
• Hot electron production can be estimated as well.
• DEPLETE can indicate regions where kinetic simulations may be especially illuminating.