INTERPLAY OF ELECTRON TRAPPING AND INHOMOGENEITY IN RAMAN SCATTERING

David J. Strozzi[1*], A. Bers[1], E. A. Williams[2], A. B. Langdon[2]

[1] *Massachusetts Institute of Technology*
Cambridge, MA 02139

[2] *Lawrence Livermore National Laboratory*
Livermore, CA 94550

Work supported by US Dept. of Energy

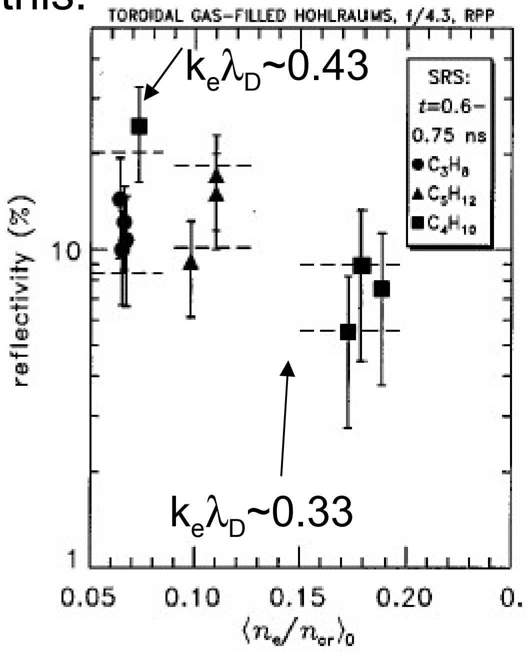
Anomalous Absorption 2005
Fajardo, Puerto Rico
Poster RP14
30 June 2005

---

[*] dstrozzi@mit.edu. Poster available at web.mit.edu/dstrozzi/www/

Is Stimulated Raman Scattering (SRS) a concern in ICF ignition plasmas?

- Linear theory: SRS convective, needs long plasmas to grow due to Landau damping. Recent experiments[1] and reduced PIC simulations[2] contradict this.



$k_e\lambda_D$~0.43

$k_e\lambda_D$~0.33

[from ref. 1]

- ELVIS: 1-D Vlasov-Maxwell solver for studying SRS.

- SRS from homogeneous plasma: bursty reflectivity,
  >> linear theory, f_e shows vortices and flattening.

- Electron trapping:
  enhances SRS by reducing Landau damping;
  nonlinearly shifts plasma wave frequency

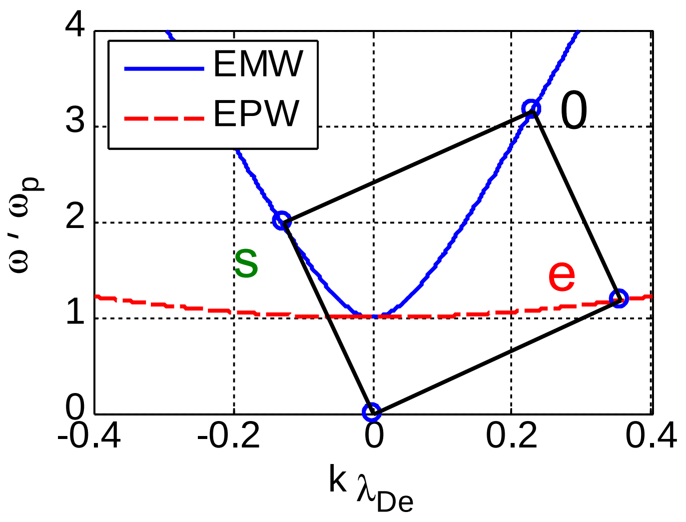
- Driven plasma wave:
  response to fixed force reveals nonlinear dielectric properties

- Inhomogeneity:
  wavevector mismatch interacts with nonlinear shift from trapping

…maybe!

[1] J. C. Fernández et al. *Phys Plasmas* **7**, 3743 (2000).
[2] H. X. Vu, D. F. DuBois, B. Bezzerides. *Phys Plasmas* **9**, 1745 (2002).

Raman scatter couples a pump light wave (0) to a Scattered Light Wave (s) and Plasma Wave (e)



ω and *k* (energy, momentum) matching

$$\omega_0 = \omega_s + \omega_e \qquad \vec{k}_0 = \vec{k}_s + \vec{k}_e$$

Coupled-Mode Equations

$$D_0 a_0 = K a_s a_e$$
$$D_s a_s = -K a_0 a_e^*$$
$$D_e a_e = -K a_0 a_s^*$$

$$D_i = \partial_t + \vec{v}_{gi} \cdot \nabla + \gamma_i$$
$$\vec{v}_{gi} = \text{group vel.}$$
$$\gamma_i = \text{damping}$$

0 $$(\partial_{tt} - c^2\nabla^2 + \omega_p^2)\vec{V}_0 = -\omega_p^2 \frac{n_e}{n_0} \vec{V}_s$$

s $$(\partial_{tt} - c^2\nabla^2 + \omega_p^2)\vec{V}_s = -\omega_p^2 \frac{n_e}{n_0} \vec{V}_0$$

e $$(\partial_{tt} - 3v_{Te}^2\nabla^2 + \omega_p^2)n_e = n_0 \nabla^2 \vec{V}_0 \cdot \vec{V}_s$$

Light waves driven by current density ~ n_e V

plasma wave driven by light-wave vxB force

wave operator

parametric coupling

Instability Thresholds vary strongly w/ γ_e (Landau damping)

$$E_i \sim \underset{\text{Slowly-varying amplitude}}{a_i(x,t)} \exp i\,(k_i \underset{\text{Rapid oscillation}}{x} - \omega_i t) + \text{c.c.}$$

Slowly-varying amplitude   Rapid oscillation

Instability threshold: $\gamma_0 > \sqrt{\gamma_s \gamma_e} \quad \rightarrow \quad I_0 > I_{\text{con}}$

Absolute instability threshold:

$$\gamma_0 > \frac{1}{2}\sqrt{|v_{gs} v_{ge}|}\left(\frac{\gamma_s}{|v_{gs}|} + \frac{\gamma_e}{|v_{ge}|}\right) \quad \rightarrow \quad I_0 > I_{\text{abs.}}$$

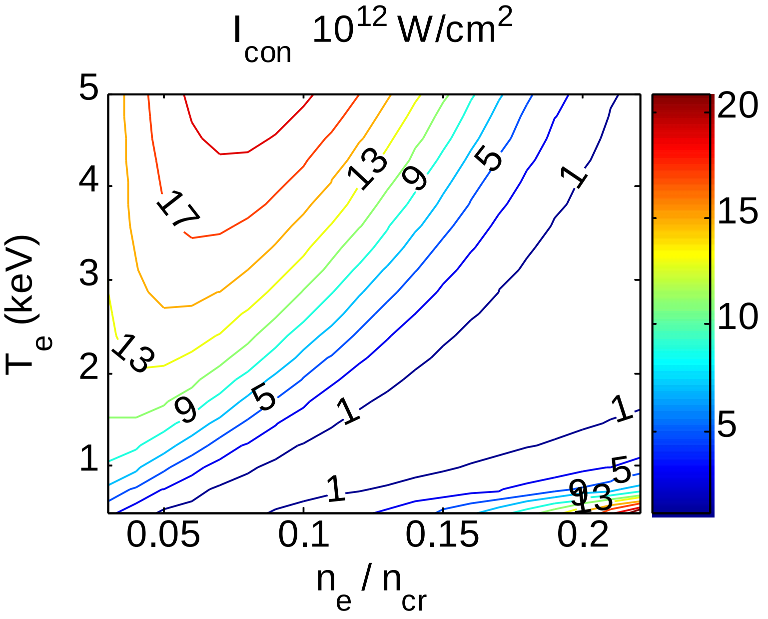
Linear (a_0 = const.) temporal growthrate:

$$\gamma = -\frac{1}{2}(\gamma_s + \gamma_e) + \sqrt{\gamma_0^2 + \frac{1}{4}(\gamma_s - \gamma_e)^2}$$

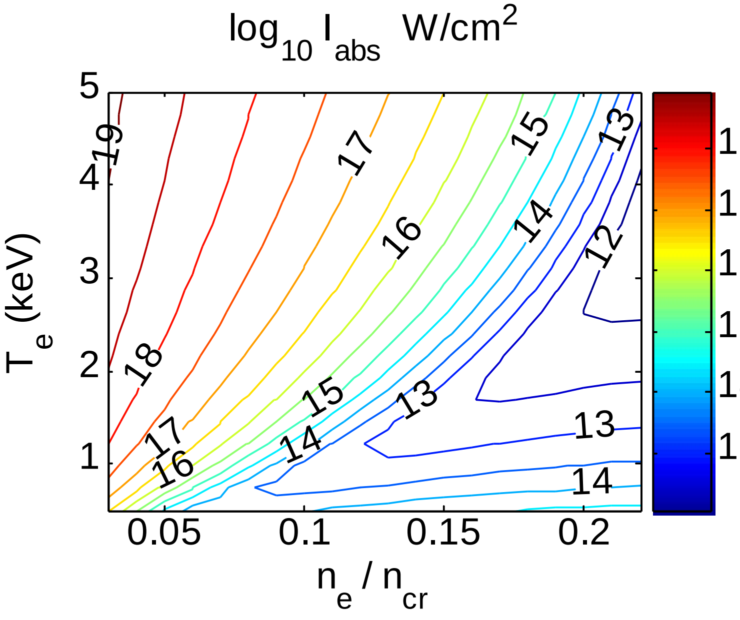
$$\gamma_0 = \frac{\omega_p}{4\sqrt{\omega_s \omega_e}} k_e v_{os,0}$$

$$= 2.0 \frac{\omega_p}{\sqrt{\omega_s \omega_e}} k_e \lambda_0 \sqrt{I_{0,15}} \quad (1/\text{ps})$$

λ_0=351 nm, 50-50 H-He T_i=T_e/3

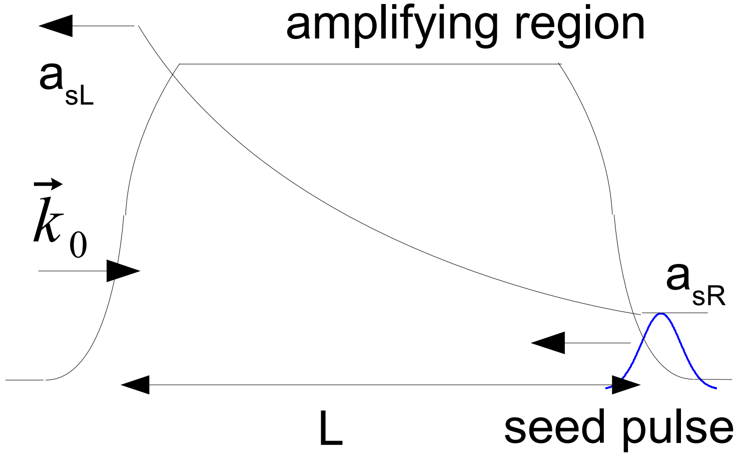


NIF I_0: beam avg ~ 5 10^{14} speckle ~ 5 10^{15}



Strong Landau damping gives mild convective gain

Convective Steady State

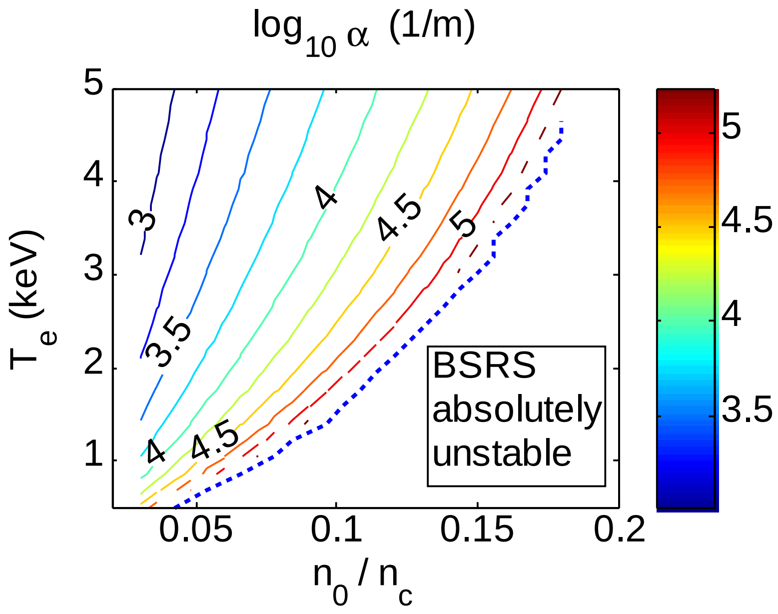


$$a_{sL} = a_{sR} e^{G} \qquad G = \alpha L$$

$$\text{Strong damping limit}: |\alpha_e a_e| \gg |\partial_x a_e|$$

$$\alpha \approx \frac{\alpha_0^2}{\alpha_e}$$

$$\alpha_0 \approx \frac{\gamma_0}{\sqrt{v_{gs} v_{ge}}}$$



I_0 = 2E15 W/cm² λ_0 = 351 nm

Linear Theory Predicts:

In ICF hohlraums, BSRS is a convective instability needing long plasmas to grow due to strong Landau damping

ELVIS: EuLerian Vlasov Integrator with Splines
1-D Vlasov-Maxwell Solver

[D. J. Strozzi, M. M. Shoucri, A. Bers, *Comp Phys Comm* **164**/1-3 (2004)]
[A. Ghizzo, P. Bertrand, M. M. Shoucri *et al.*, *J Comp Phys* **90** (1990)]

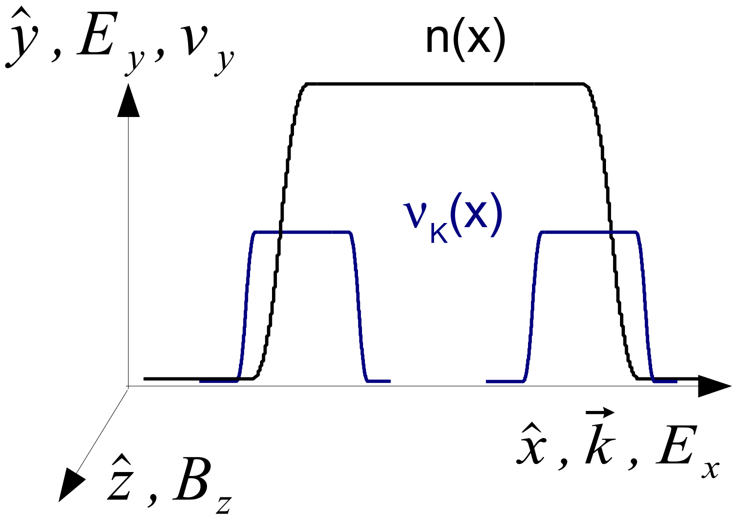
- Kinetic equation in *x*:

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}+(E_x+v_y B_z)\frac{\partial f}{\partial p}=\boxed{-\nu_K(x)(f-n\hat{f}_{0K})}$$

Krook operator

- Gauss' Law:

$$\partial_x E_x=e\epsilon_0^{-1}(Z_i n_i-n_e)$$



- Transverse flow: cold collisionless fluid

$$m_e\partial_t v_y=-eE_y$$

- Transverse Maxwell Fields: linearly polarized in *y*

$$E^{\pm}\equiv E_y\pm cB_z \qquad (\partial_t\pm c\partial_x)E^{\pm}=-\epsilon_0^{-1}J_y \qquad E^{\pm}=\text{ right, left moving}$$

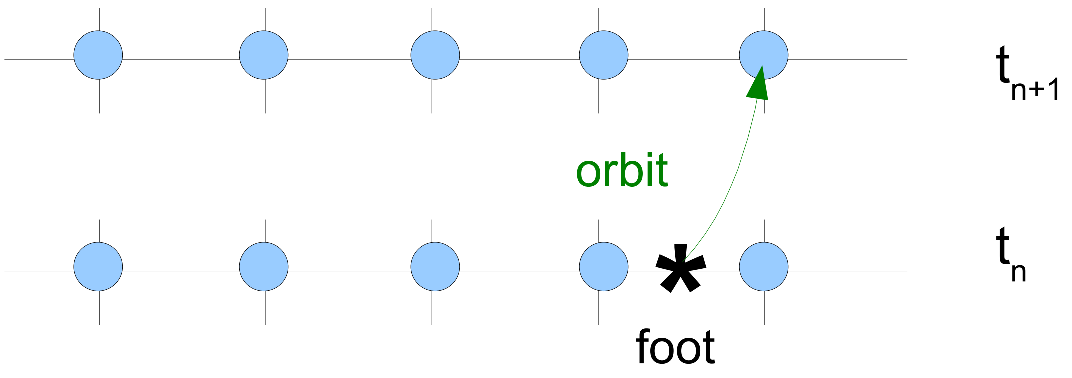
Numerical Algorithm: Vlasov vs. PIC

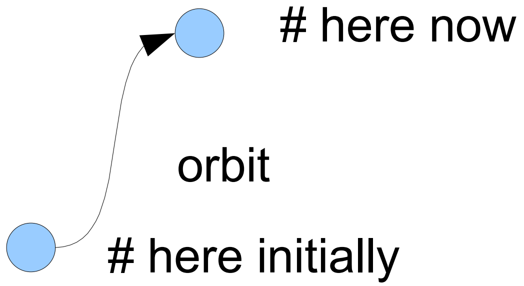
Continuum ("Vlasov," Eulerian) method: Treat *f* as phase-space fluid; no discreteness
Particle-in-Cell (PIC, Lagrangian) method: Follow macro-particles; discreteness effects

Continuum methods have much lower numerical noise and can study small-amplitude dynamics better; however, they require enormous resources to grid, e.g., 6-D phase space.

Vlasov eqn: *f* constant along orbits $X(t)$, $P(t)$

$$\frac{dX}{dt} = V \qquad \frac{dP}{dt} = F$$





new f at gridpoint =
old f at orbit foot

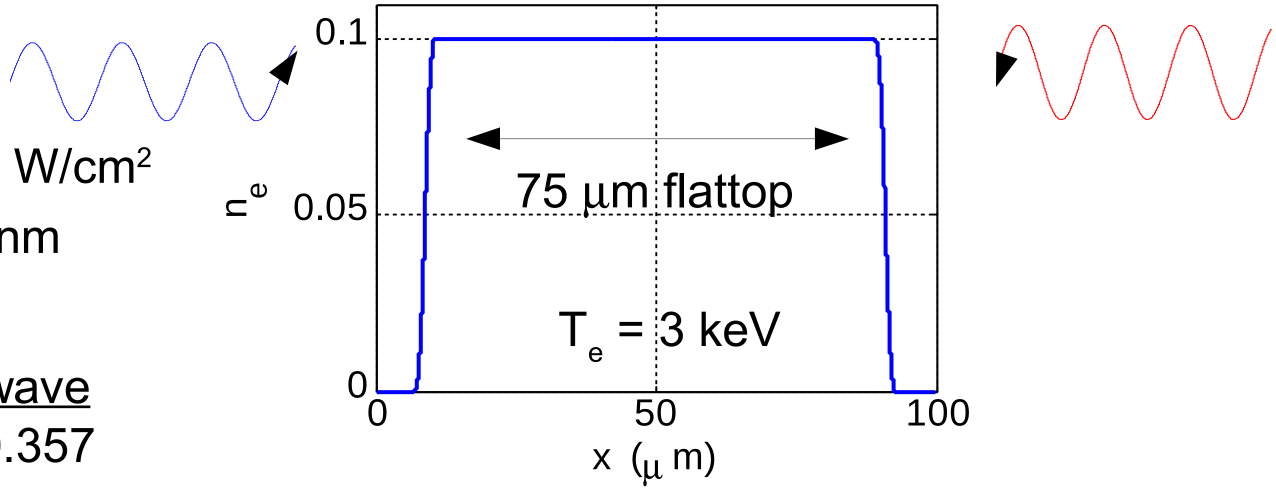
Operator Splitting: [C. Z. Cheng, G. Knorr, *J Comp Phys* **22** (1976)]

$$1. \quad (\partial_t + v\partial_x)f = 0 \quad \rightarrow \quad f^*(x,p) = f(x - v\,dt, p, t_{n+1})$$

$$2. \quad (\partial_t + F\partial_p)f = 0 \quad \rightarrow \quad f(x,p,t_{n+1}) = f^*(x, p - F\,dt)$$

Use cubic spline interpolation for f at foot → tri-diagonal system

Homogeneous plasma: SRS well above linear gain



pump

I_0 = 2E15 W/cm^2

λ_0 = 351 nm

seed

I_s = 2E10 W/cm^2

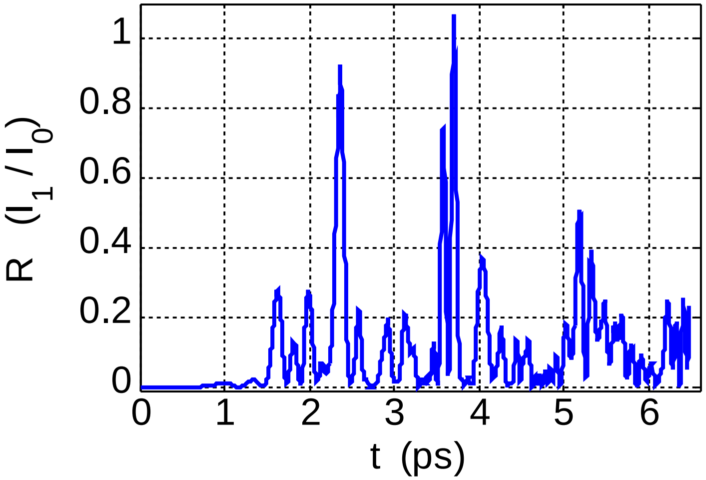
λ_s = 574 nm

plasma wave

$k_e\lambda_{De}$ = 0.357

ν_e = $0.031\omega_e$

Reflectivity at left edge
avg. (1:6 ps) = 13%



linear theory:

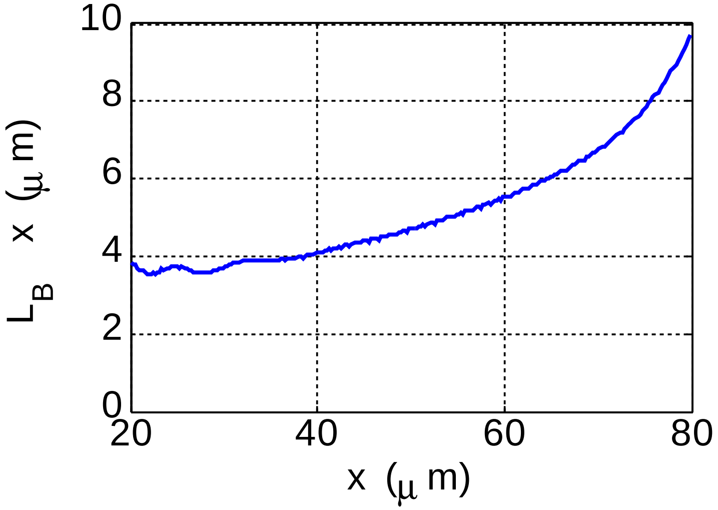
R = 0.0173%

α^{-1} = 52.6 μm

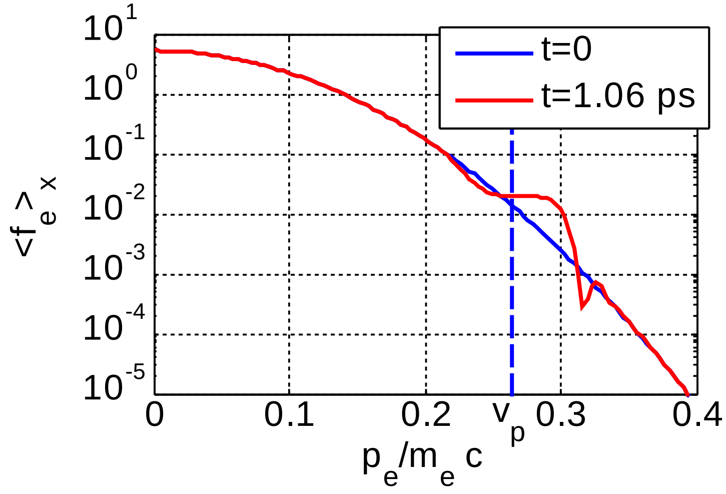
Run SRS is bursty, no steady state

Homogeneous plasma: trapping happens, *f*$_e$ flattened

t (ps) = 0.825



Spatially-averaged *f*$_e$

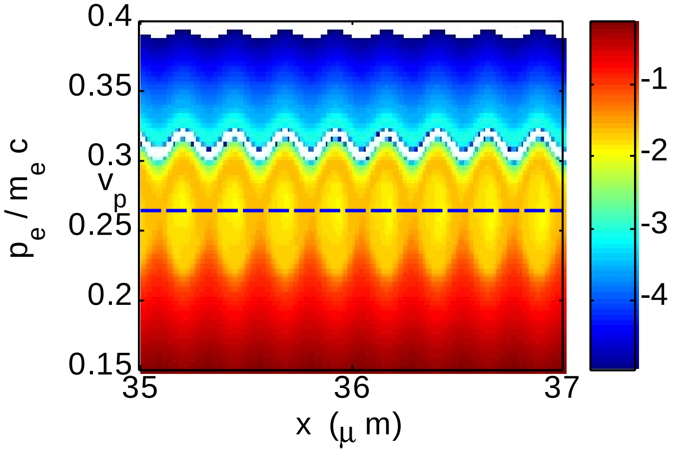


$$\omega_B = \omega_p \left(\frac{\delta n}{n_0} \right)^{1/2}$$

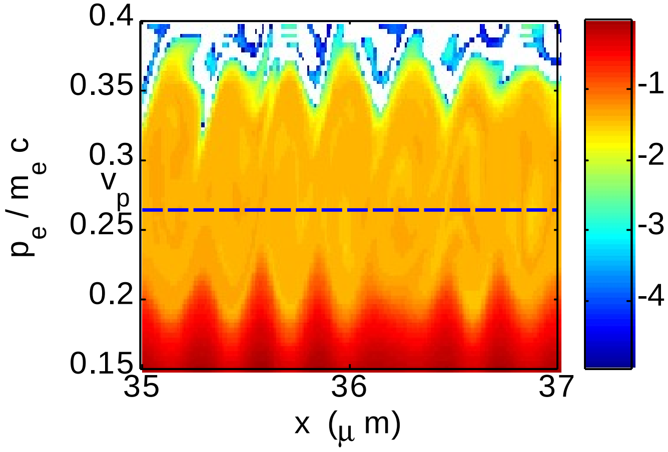
$$k_B = \frac{\omega_B}{v_{pe}}$$

$$L_B = \frac{2\pi}{k_B}$$

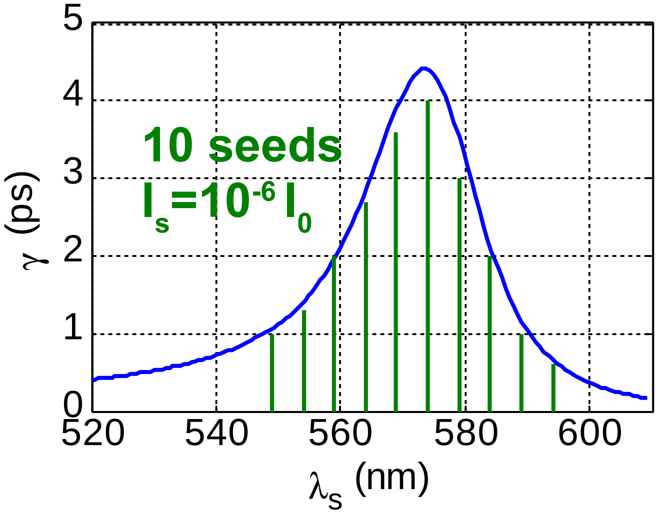
$\log_{10} f_e$ t = 1.06 ps



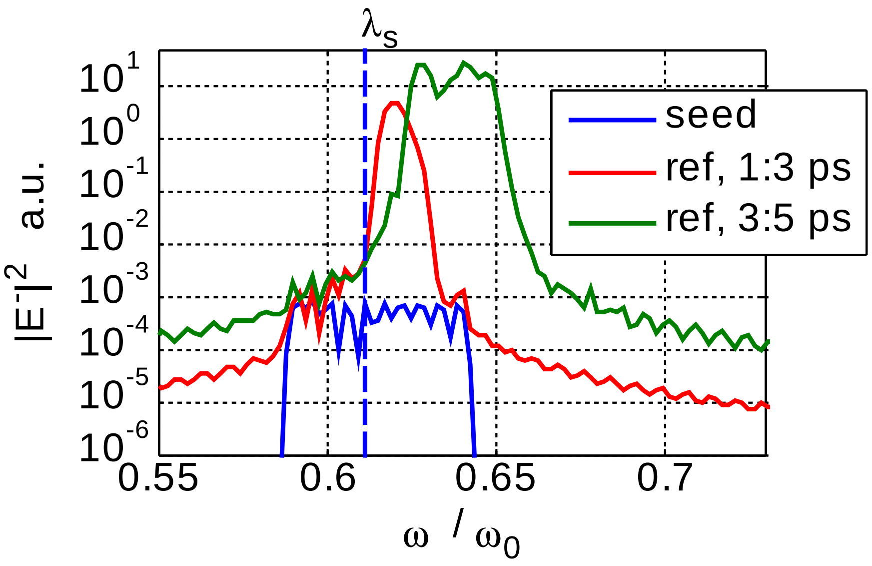
$\log_{10} f_e$ t = 1.77 ps



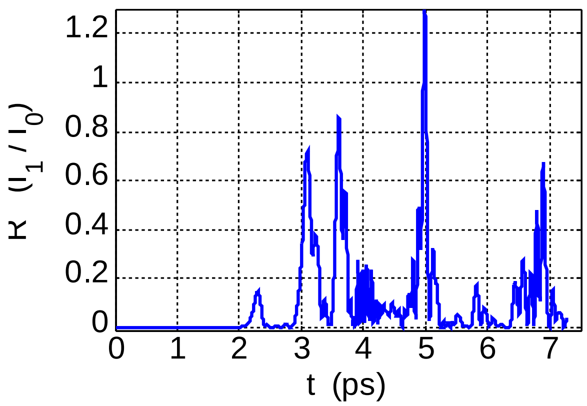
Seed bandwidth: trapping at first by fast-growing mode

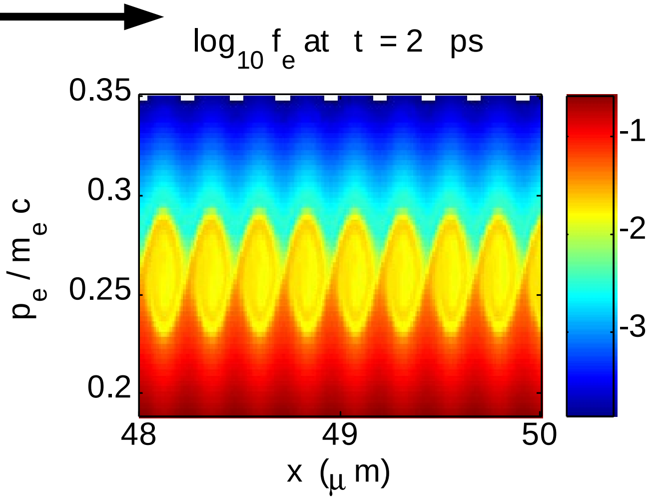


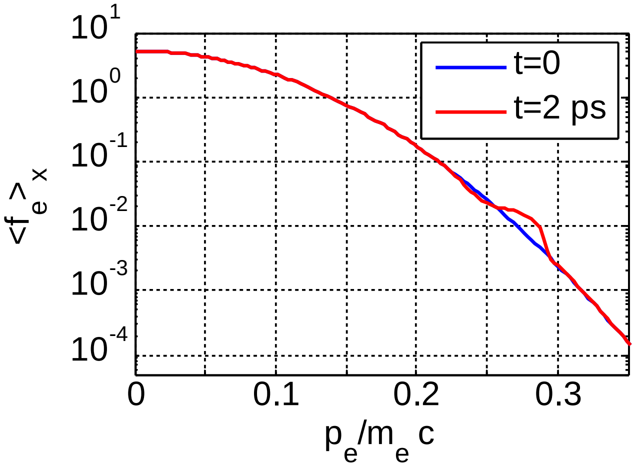
$$\varepsilon(k_e,\omega_e) = \frac{\chi\ k_e^2 v_{os,0}^2\ /4}{c^2(k_0-k_e)^2+\omega_p^2-(\omega_0-\omega_e)^2}$$



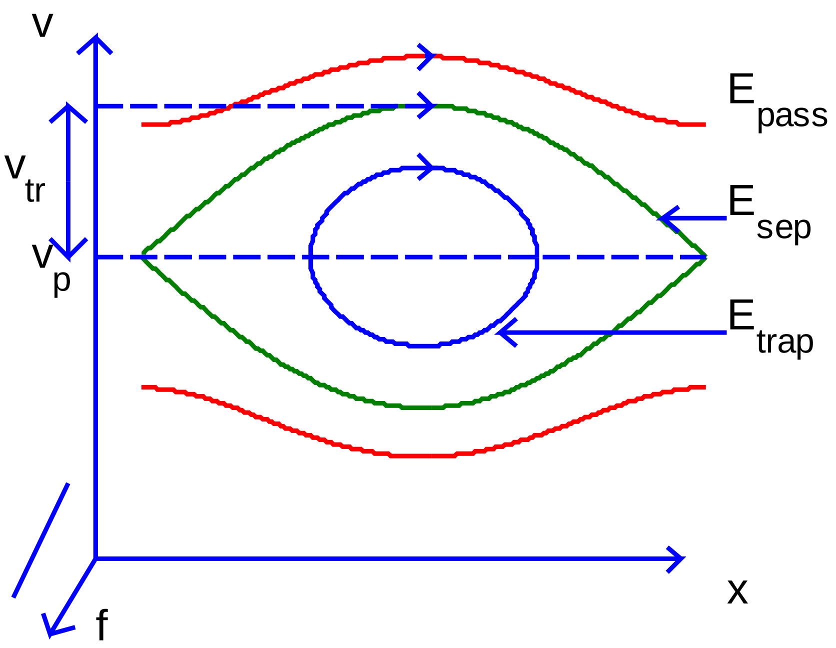
avg R (2:7 ps) = 14%







Electron Trapping Reduces Landau Damping



$$\omega_B = \left(\frac{ekE_0}{m_e}\right)^{1/2} = \omega_p\left(\frac{\delta n}{n_0}\right)^{1/2}$$

$$v_{tr} = 2\left(\frac{eE_0}{m_e k}\right)^{1/2} \approx 2v_p\left(\frac{\delta n}{n_0}\right)^{1/2}$$

$$k_B = -\frac{\omega_B}{v_p} \approx -k\left(\frac{\delta n}{n_0}\right)^{1/2}$$

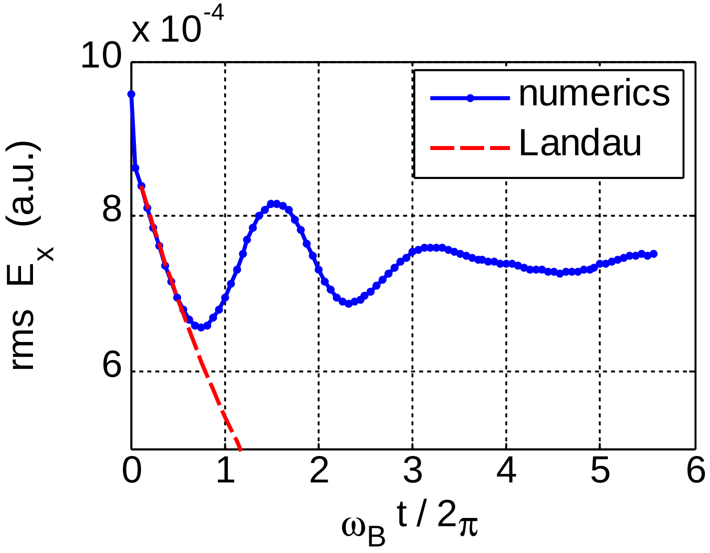
Periodic elestrostatic run (no light waves)

$\delta n/n_0 = 0.005$ $\quad k\lambda_D = 0.273$

$T_e = 1\text{keV}$ $\quad n_0 = 5E26\ \text{m}^{-3}$

$\omega_B = 0.0707\omega_p$

Damping reduced once trapped electrons bounce [T. O'Neil, *Phys Fluids* **8**, 1965]

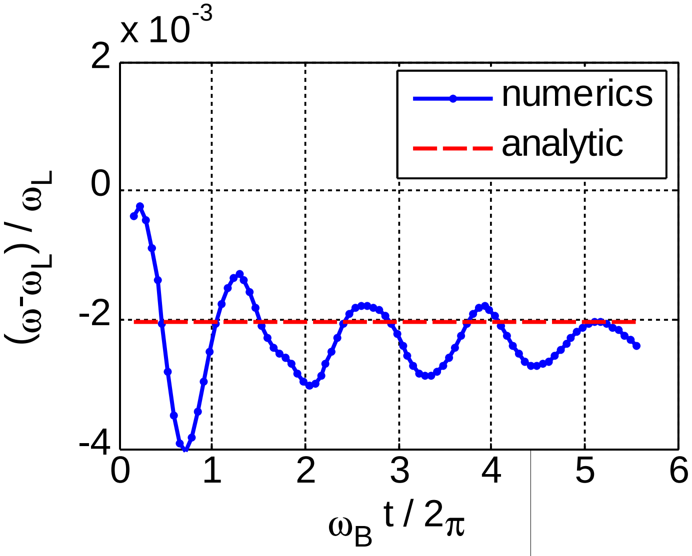


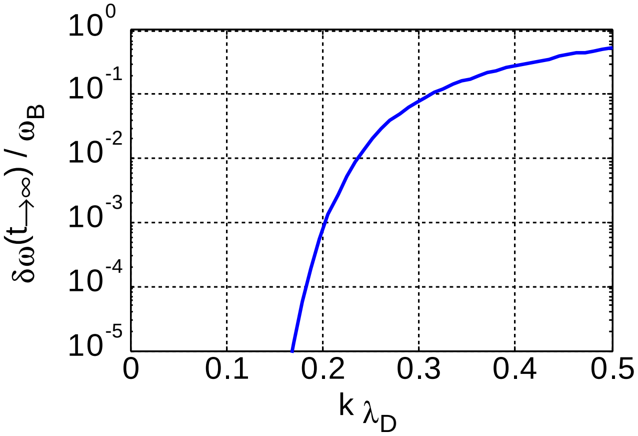
Trapping Also Downshifts Wave Frequency

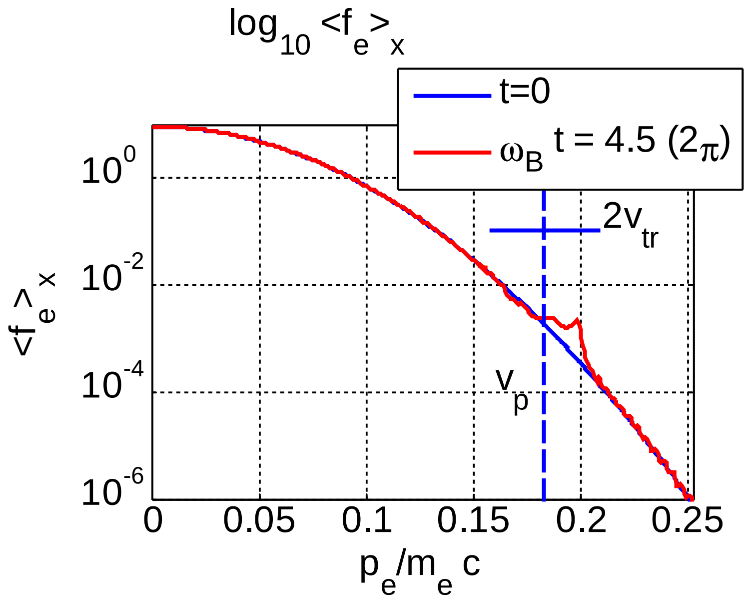
$$\frac{\delta\omega(t \gg \tau_B)}{\omega_B} = -1.645\left(\frac{\omega_p}{k}\right)^3 \frac{\hat{f}_0''}{\omega_L \partial\varepsilon_r/\partial\omega_L}$$

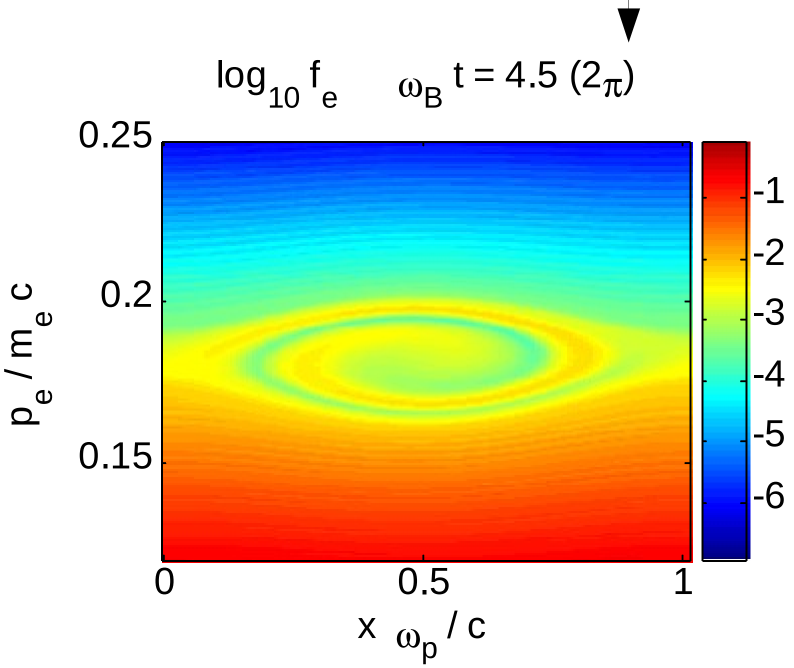
[G. J. Morales and T. M. O'Neil. *PRL* **28**, 417 (1972)]

f_0 Maxwell. → $1.856\dfrac{2\zeta^2-1}{Z_r''(\zeta)}e^{-\zeta^2}$ $\qquad \zeta = \dfrac{v_p}{v_T\sqrt{2}}$

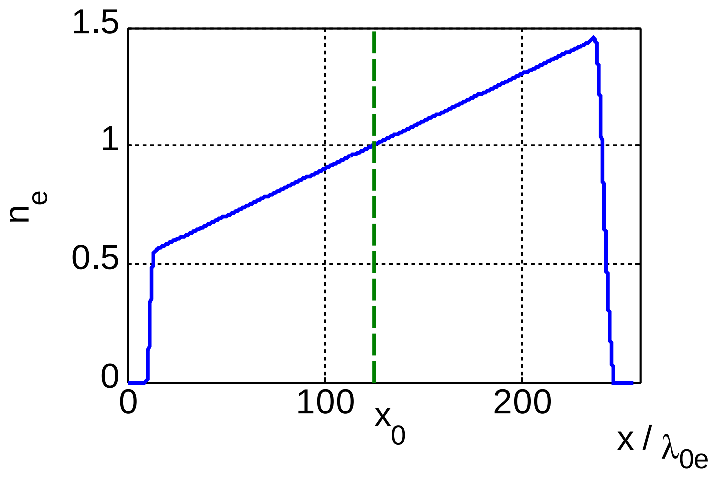








Plasma Response to External Driver: Theory



full lin nonlin

$$\varepsilon = \varepsilon_l + \delta\varepsilon$$

linear mode

$$\varepsilon_l(k_{el}) = 0$$

$$k_{el} = k_{elr} + i\sigma_{el}$$

nonlin. mode

$$\varepsilon(k_{en}) = 0$$

$$k_{en} = k_{el} + \delta k_e$$

$$\delta k_e = -\frac{\delta\varepsilon}{\partial\varepsilon_l/\partial k_{el}}$$

Driver:

$$\phi_0 = \tilde{\phi}_0 \exp i(k_0 x - \omega_0 t) + c.c.$$

Steady State:

$$\varepsilon(\omega_0, k_0 - i\partial_x)\tilde{\phi} = \tilde{\phi}_0$$

strong damping limit ($\partial_x = 0$):

$$\tilde{\phi} = \frac{\tilde{\phi}_0}{\varepsilon_l(k_0) + \delta\varepsilon(k_0)}$$

$$\approx \frac{\tilde{\phi}_0 \; / \; \partial\varepsilon_l/\partial k_l}{k_0 - k_{lr} - i\sigma_l - \delta k_{nl}}$$

Including advection:

$$\varepsilon(k_e - i\partial_x) \approx \varepsilon(k_e) - i\frac{\partial\varepsilon}{\partial k_e}\partial_x$$

$$\approx \frac{\partial\varepsilon_l}{\partial k_{el}}(-i\partial_x + \kappa(x) - i\sigma_{el} - \delta k_e)$$

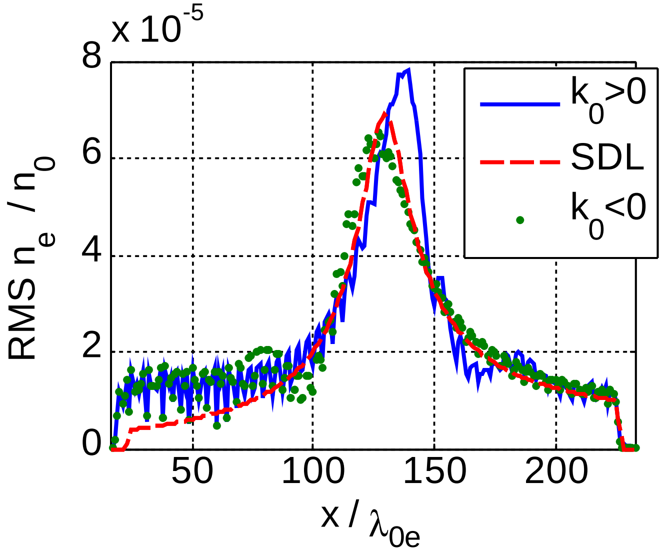
advection mismatch damping shift

$\kappa = k_0 - k_{lr}$

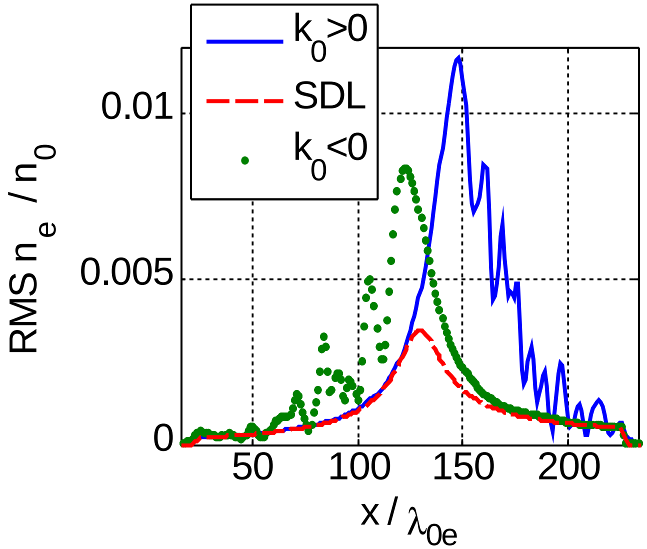
Plasma Response to External Driver: Simulation

$k_0>0$, $k_0<0$: EPW propagates to the right, left

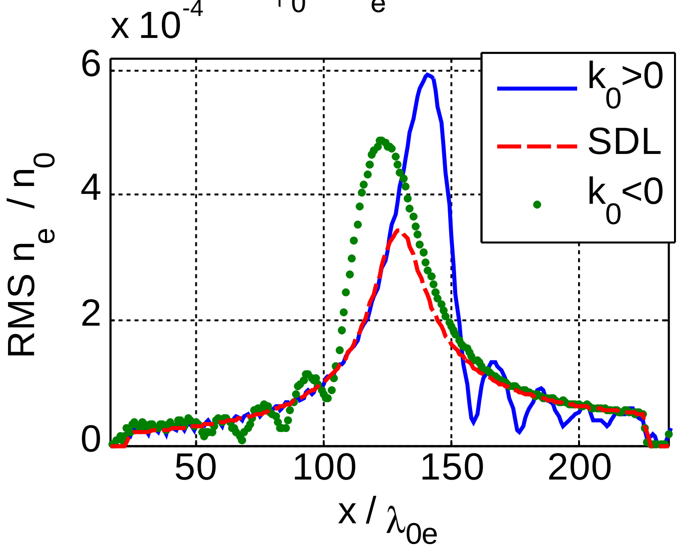
$e\phi_0 / T_e = 4E-5$



$e\phi_0 / T_e = 2E-3$

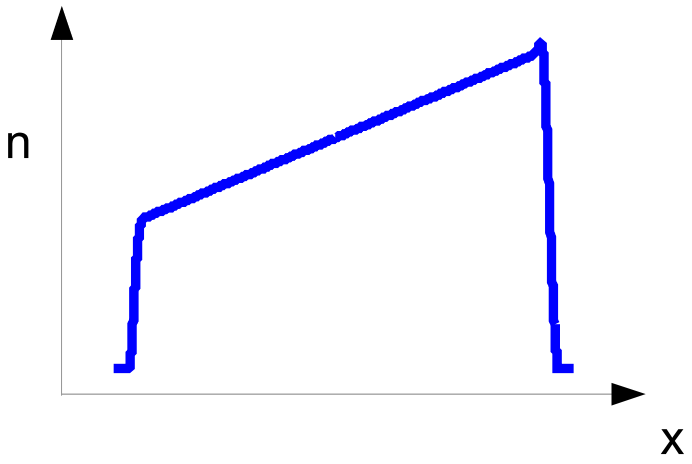


$e\phi_0 / T_e = 2E-4$



- Advection gives left/right asymmetry?
- What is the nonlinear k shift, damping rate for a driven wave?
- Use this model for $\delta\varepsilon$ in full SRS problem

Inhomogeneous plasma: *k*'s vary with *x*



$$\begin{aligned} \omega_e &= \omega_0 - \omega_s && \text{matching} \\ k_e(x) &= k_0(x) - k_s(x) && \text{beat mode (not n.m.)} \\ \kappa(x) &\equiv k_e(x) - k_{elr}(x) && \text{mismatch} \end{aligned}$$

full lin nonlin

$$\varepsilon = \varepsilon_l + \delta\varepsilon$$

linear mode

$$\varepsilon_l(k_{el}) = 0$$

$$k_{el} = k_{elr} + i\sigma_{el}$$

nonlin. mode

$$\varepsilon(k_{en}) = 0$$

$$k_{en} = k_{el} + \delta k_e$$

$$\delta k_e = -\frac{\delta\varepsilon}{\partial\varepsilon_l/\partial k_{el}}$$

Steady state, light undamped:

$$\partial_t = \nu_0 = \nu_s = 0$$

$$v_{g0}a_0' = Ka_s a_e$$

$$v_{g1}a_1' = -Ka_0 a_e^*$$

$$\varepsilon(k_e - i\partial_x, \omega_e)a_e = 2i\frac{\omega_2}{\omega_{p0}^2}\chi\, K\, a_0 a_s^*$$

$$\varepsilon(k_e - i\partial_x) \approx \varepsilon(k_e) - i\frac{\partial\varepsilon}{\partial k_e}\partial_x$$

$$\approx \frac{\partial\varepsilon_l}{\partial k_{el}}(-i\partial_x + \kappa(x) - i\sigma_{el} - \delta k_e)$$

advection mismatch damping shift

Strong damping limit: neglect plasma wave advection

$$|\partial_x a_e| \ll |(\kappa + i\sigma_{el})a_e|$$
$$\rightarrow \varepsilon(k_e - i\partial_x) \approx \varepsilon(k_e)$$

$$a_e \approx 2i\frac{\omega_e}{\omega_{p0}^2}\frac{\chi_l(k_e)}{\varepsilon_l(k_e)}Ka_0a_s^*$$

energy density W_i

action density $N_i = W_i/\omega_i = a_i a_i^*$

action flux $Z_0 = \dfrac{N_0}{N_{0L}}$

$$Z_s = -\frac{v_{gs}}{v_{g0}}\frac{N_s}{N_{0L}}$$

Linear scattered light solution ($\delta k_e = 0$)

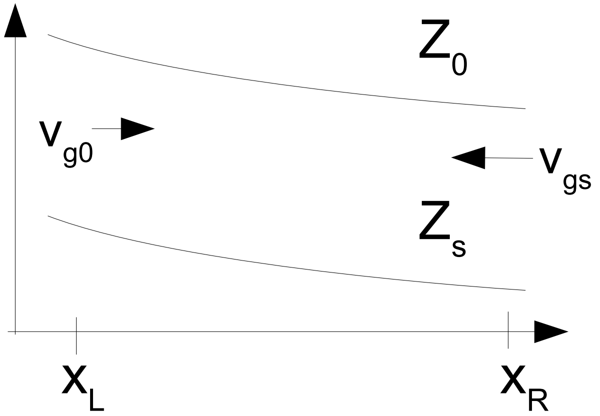
Manley-Rowe: $Z_0 - Z_s$ = const.

$$Z_s' = -\alpha_i Z_0 Z_s$$

$$\alpha = 4\frac{\gamma_0^2\omega_e}{|v_{gs}|\omega_{p0}^2}\frac{\chi_l(k_e)}{\varepsilon_l(k_e)} \qquad \alpha_i > 0$$

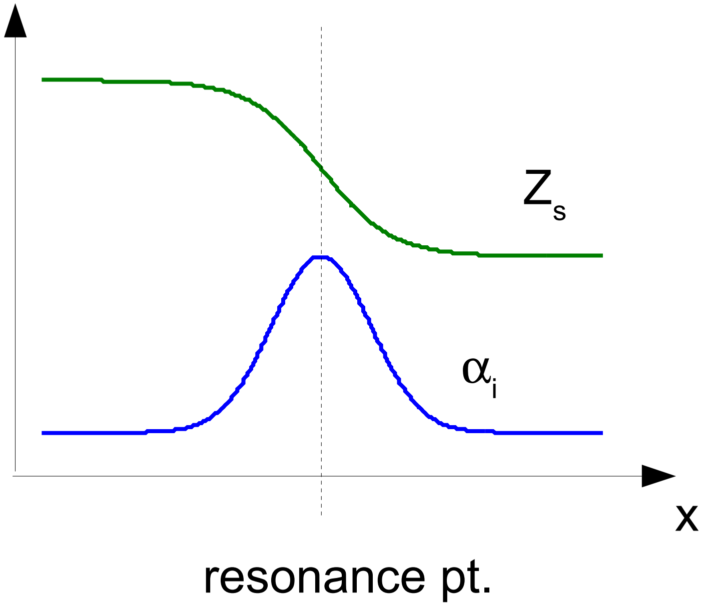
$$Z_s = \hat{Z}\Big[\Big(1 + \hat{Z}/Z_{sR}\Big)e^{-\hat{Z}G(x)} - 1\Big]^{-1}$$
$$\approx Z_{sR}\exp\hat{Z}G(x) \qquad \hat{Z} \equiv 1 - \hat{Z}_{sL}$$

$$G(x) = -\int_x^{x_R} dx'\alpha_i(x')$$



backscatter: $v_{g0} > 0$ $v_{gs} < 0$

Nonlinear shift can reduce or enhance Raman



$$Z_s' = -\alpha_i Z_0 Z_s$$

$$\alpha_i \sim \frac{1}{\sqrt{D_r^2 + D_i^2}}$$

$$D_r = \kappa - \delta k_{er} \approx \kappa' x - \delta k_{er}$$

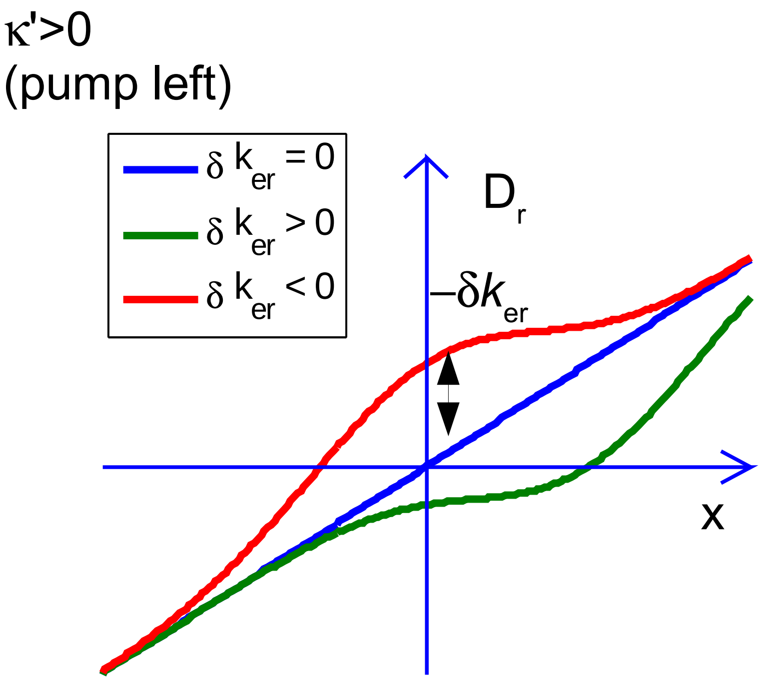
$$D_i = \sigma_{el} + \delta k_{ei}$$

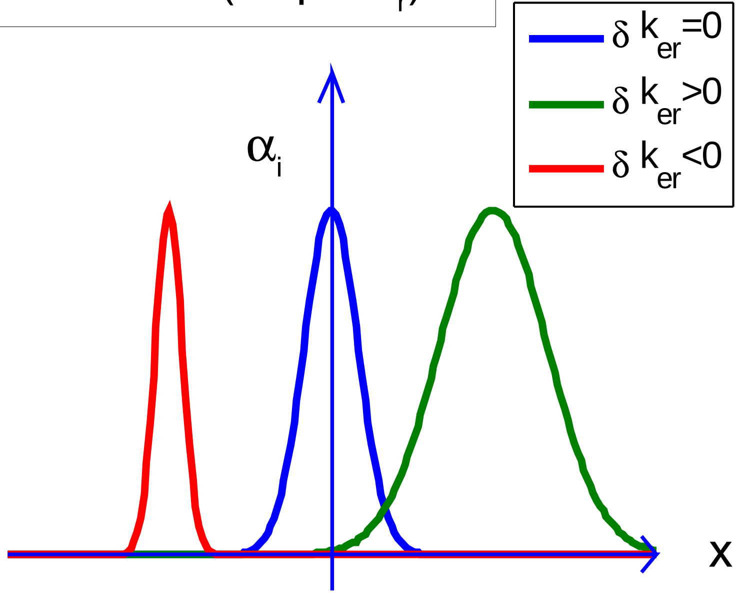
$$Z_e \sim |\alpha|^2 Z_0 Z_s$$

$$\delta k_{er} \sim Z_e^{1/4}$$

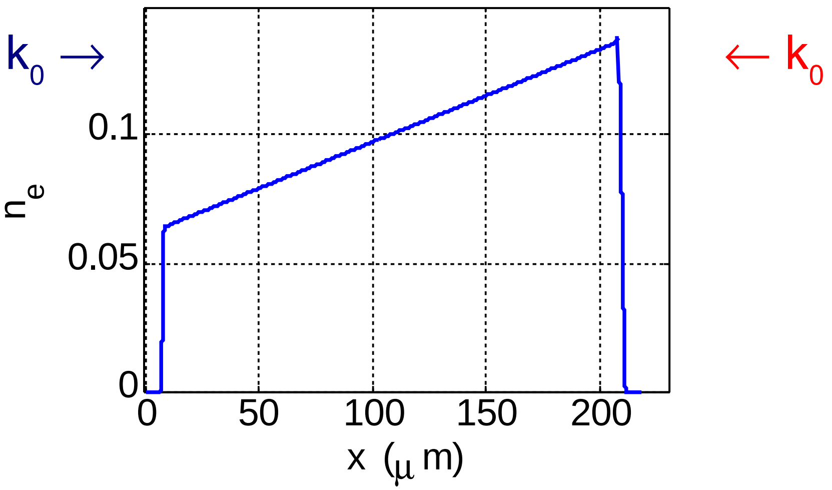
Effects of δk_e :

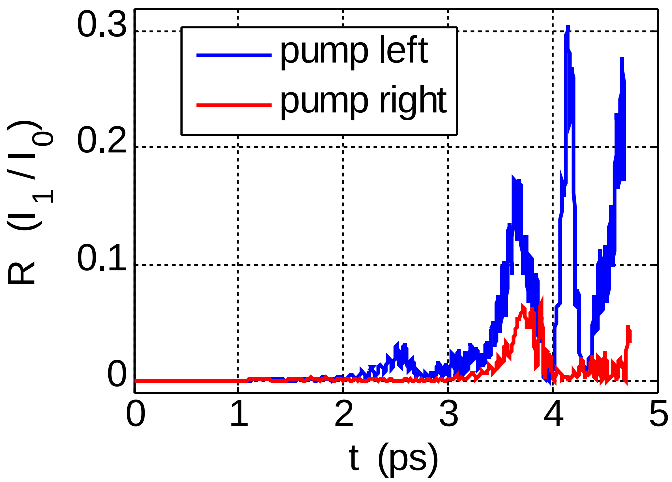
- Shifts resonance point (min. D_r)
- changes rate of passage through resonance (slope D_r)



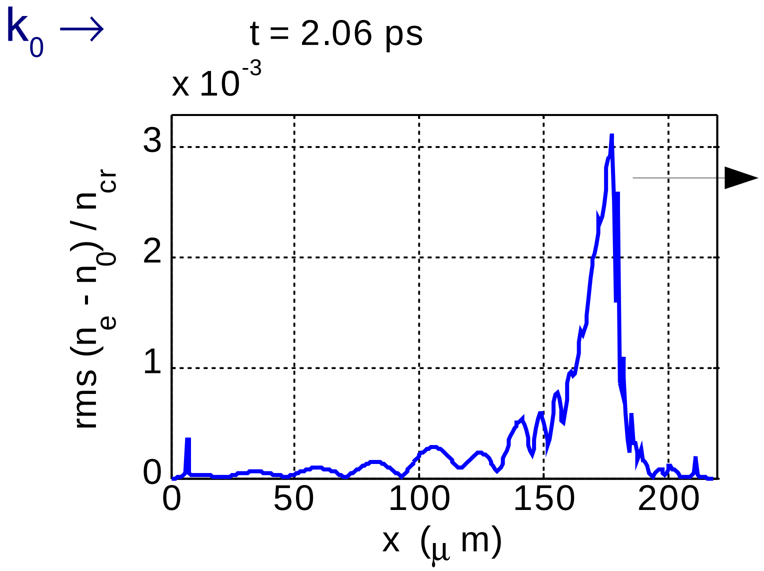


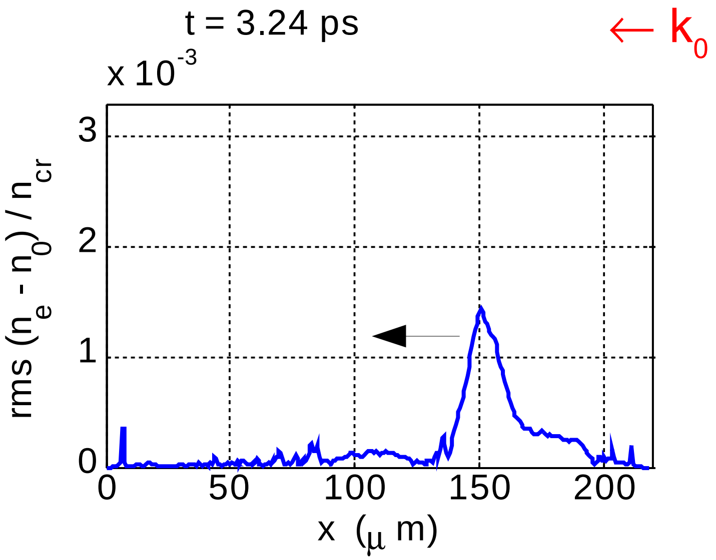
SRS in density gradient: plasma waves advect, large at edges





linear SDL Reflectivity = 1.2E-4





Summary and Future Work

Summary:

- Linear theory gives weak SRS due to high Landau damping.
- Homogeneous plasma: electron trapping reduces Landau damping, giving much larger reflectivity. This is robust against bandwidth.
- Driven plasma waves in inhomogeneous plasma show importance of advection, nonlinear shift, damping reduction; need accurate model.
- Inhomogeneous SRS: mismatch and nonlinear shift may counteract each other. Auto-resonance?

Future work:

- Sideloss out of laser speckle or beam detraps electrons. Restores Landau damping for $\gamma_{SL} > \omega_B$. ELVIS has a Krook operator to study this.
- Are ions relevant? Does the Langmuir Decay Instability of the plasma wave saturate Raman? ELVIS has option for mobile ions.

Comments/Reprints: Leave Name, Contact Info